Implicits: Intro and Tour

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Emphasis

High-level concepts, arcane details

Versatility and diversity of implicit methods

Ease in addressing certain problems

Topics

Basics

Blends

Visualization

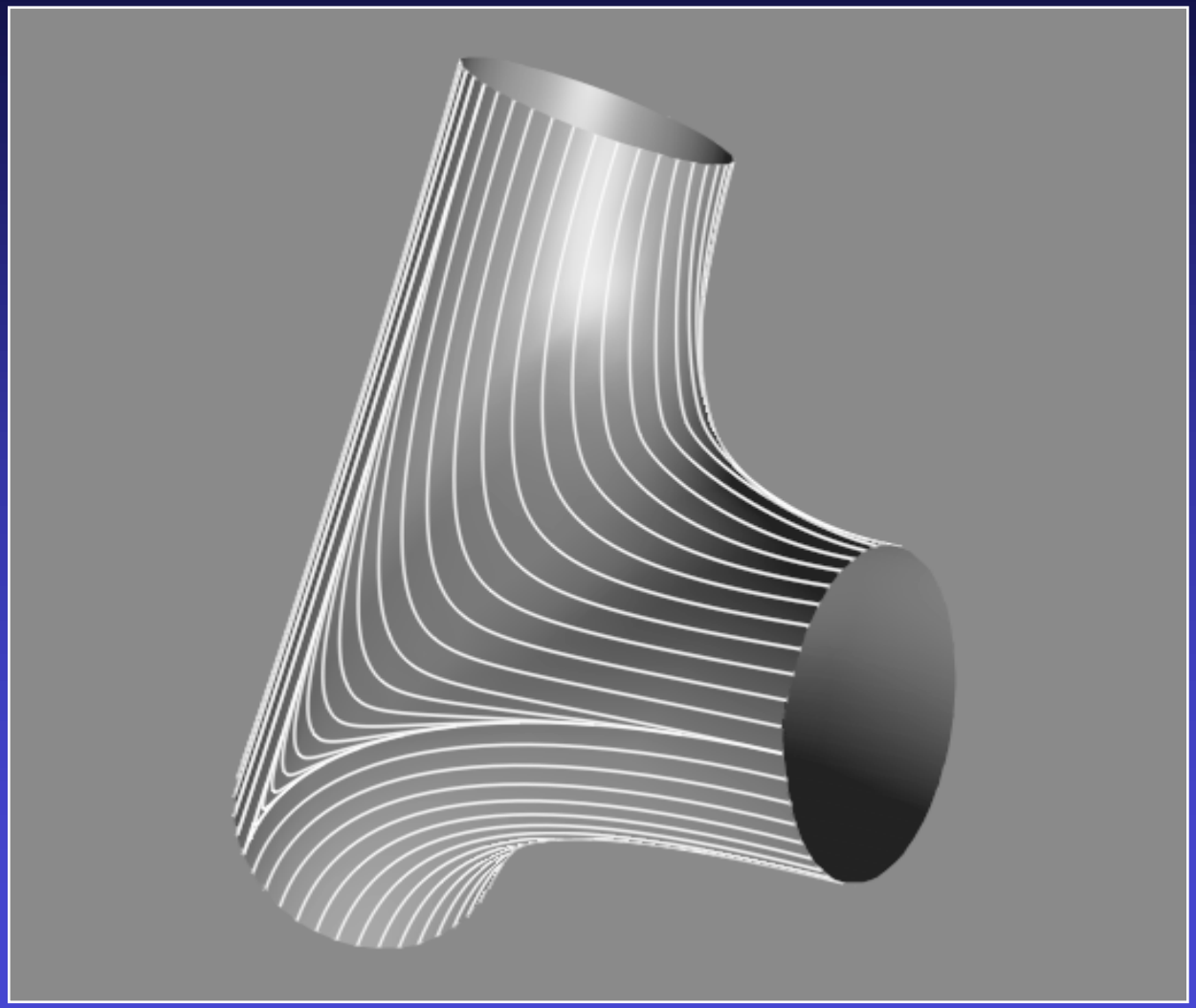
Non-Manifold Polygonization

Applications

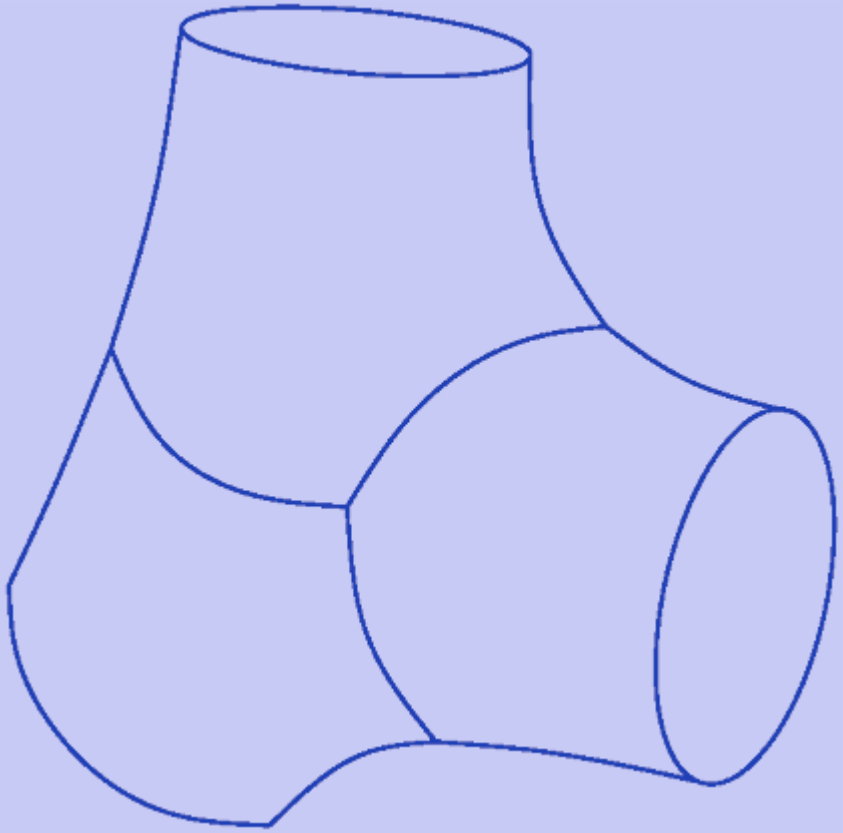
- *computing the medial axis*
- *computing deformation weights for character animation*


The Mighty Maple
1985

Lofted Ramiform



Complexity of Patch Network



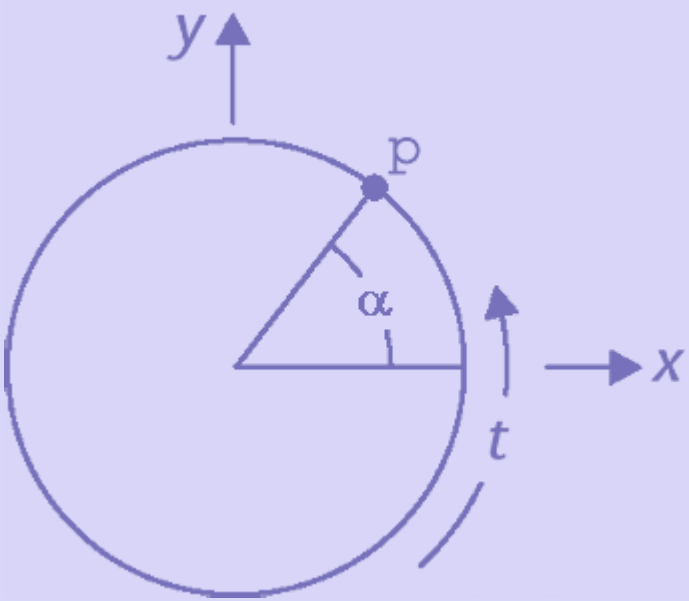
Implicit Surface

Definition

$$\{\, \boldsymbol{p} : \mathrm{f}(\boldsymbol{p}) = 0 \,\}$$

N.B. ‘such that’

Three Representations for the Unit Circle



Equiangular Parametric

(transcendental trigonometric)

$p = (\cos(\alpha), \sin(\alpha))$

Non-Equiangular Parametric

(rational trigonometric)

$p = (\pm(1-t^2) / (1+t^2),\ 2t / (1+t^2))$

Implicit

$p_x^2+p_y^2-1 = 0$

Implicit

$f(x,y,z) = 0$

characterizes volume

blends easy

inside/outside easy

point generation hard

precise control hard

Parametric

$(x,y,z) = F(u,v)$

characterizes surfaces

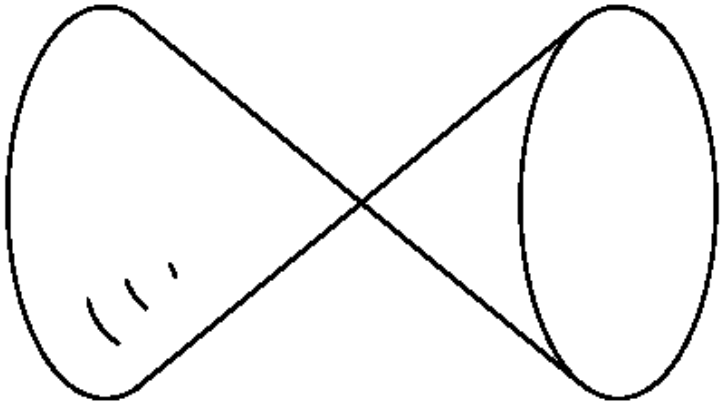
blends hard

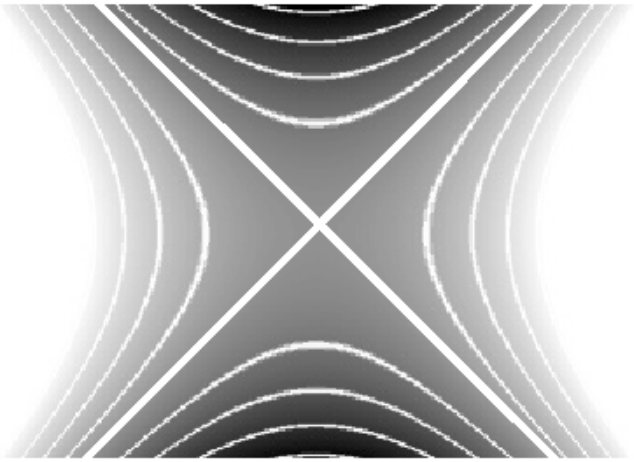
inside/outside hard

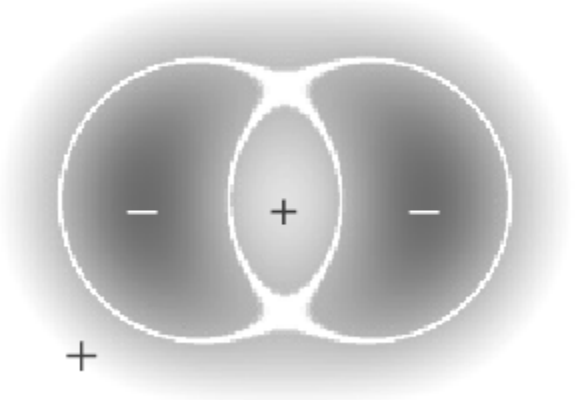
point generation easy

precise control easy

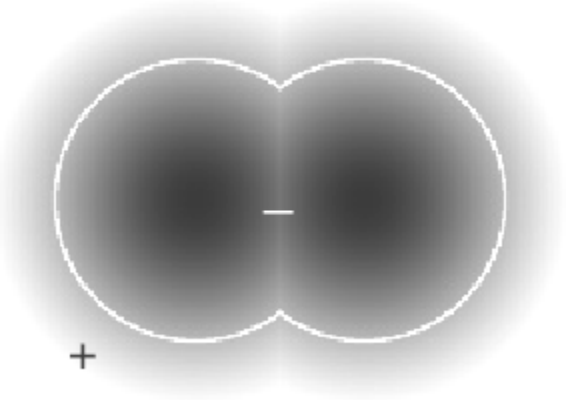
Discontinuities / Degeneracies





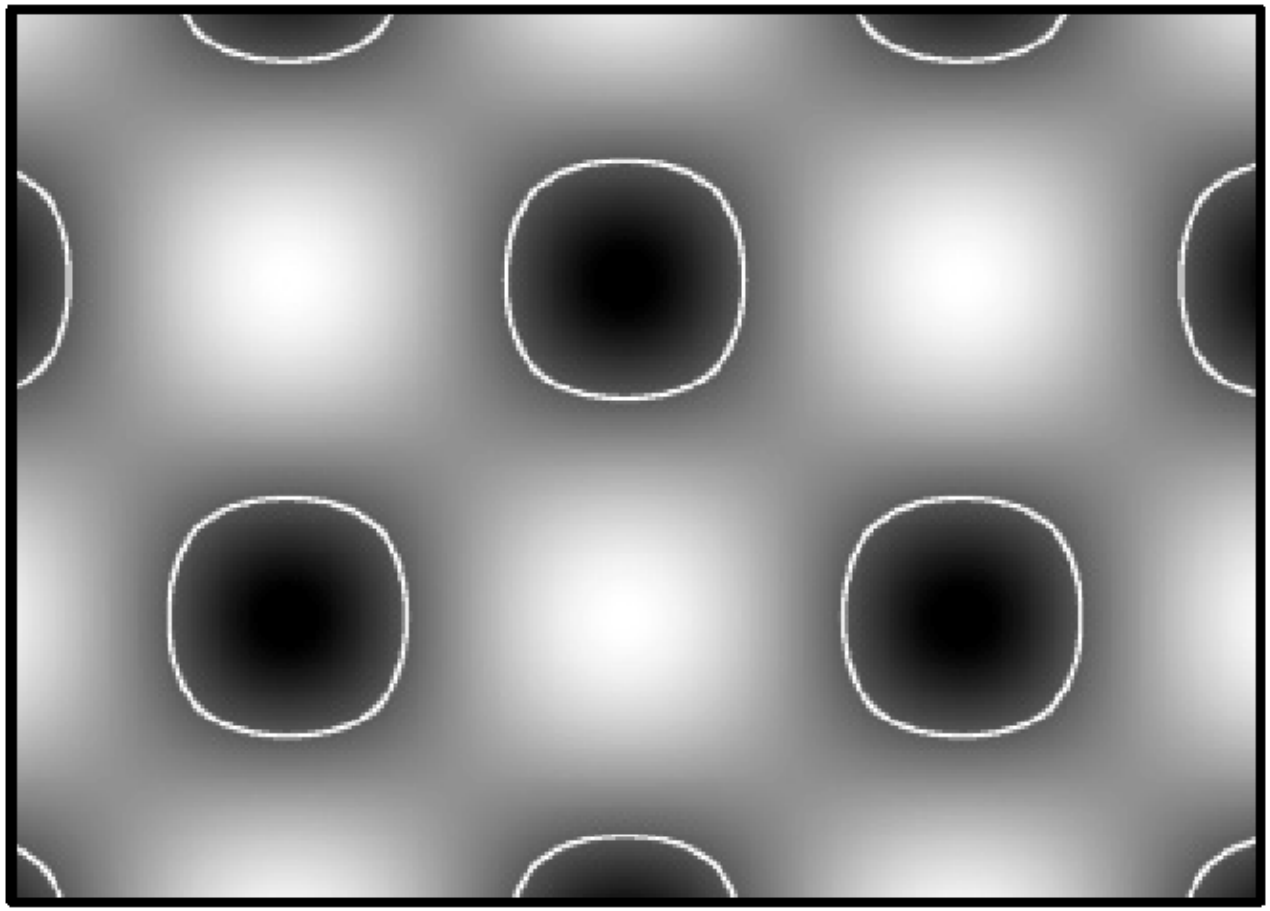


$f_1 f_2$

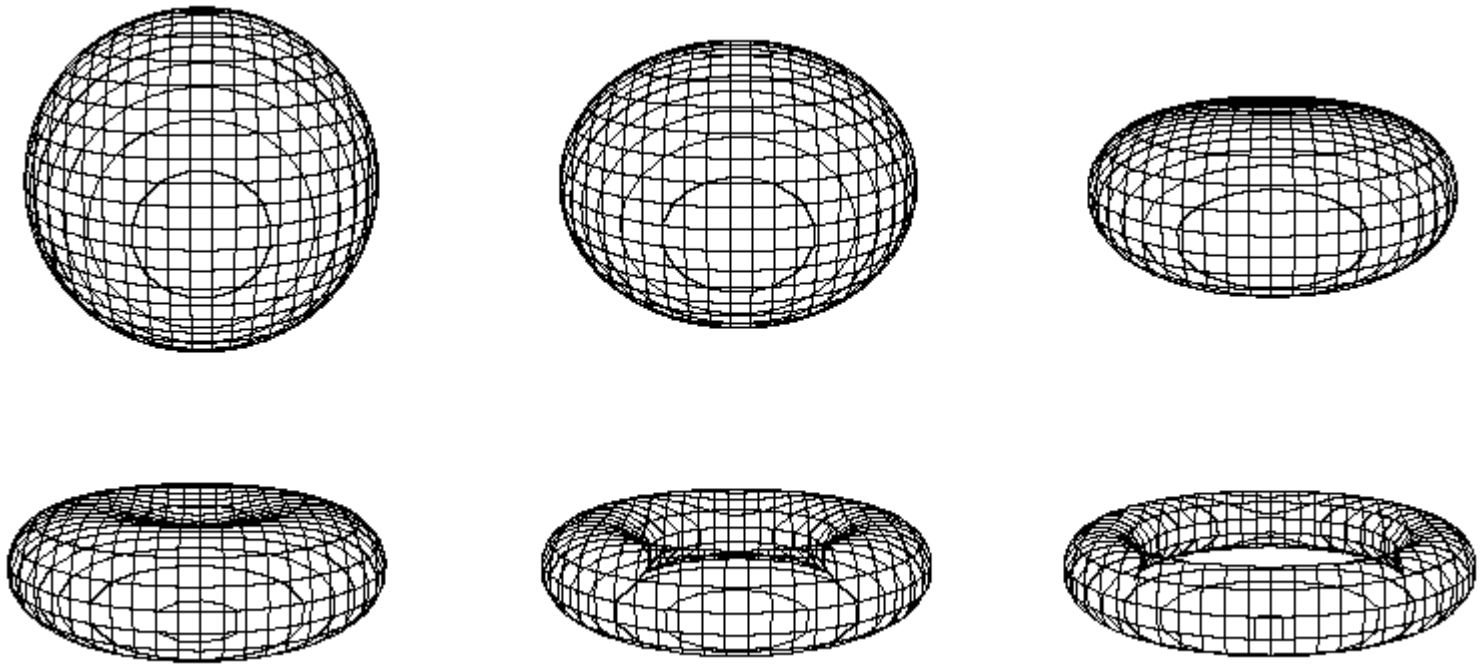


$min\ (f_1, f_2)$

Multiple Contours
from Periodic Function



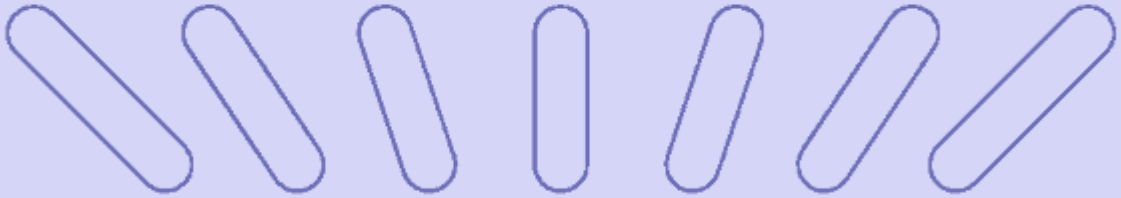
Interpolation



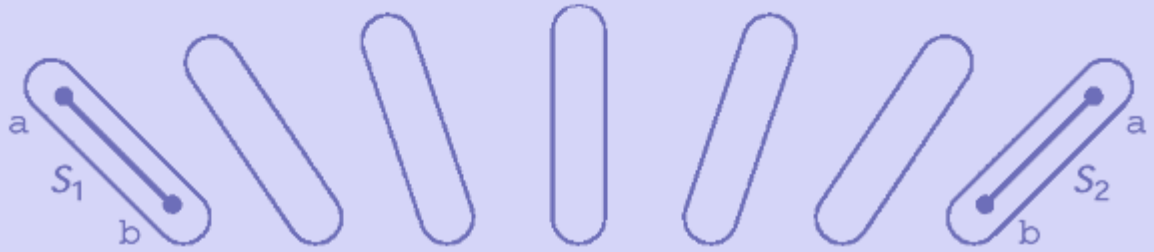
Articulation

$f(\mathrm{p}) = td(S_1, \mathrm{p}) + (1 - t)d(S_2, \mathrm{p}) - c$, where $d(S, \mathrm{p})$ is Euclidean distance between point p and segment S

$f(\mathrm{p}) = td^2(S_1, \mathrm{p}) + (1 - t)d^2(S_2, \mathrm{p}) - c$

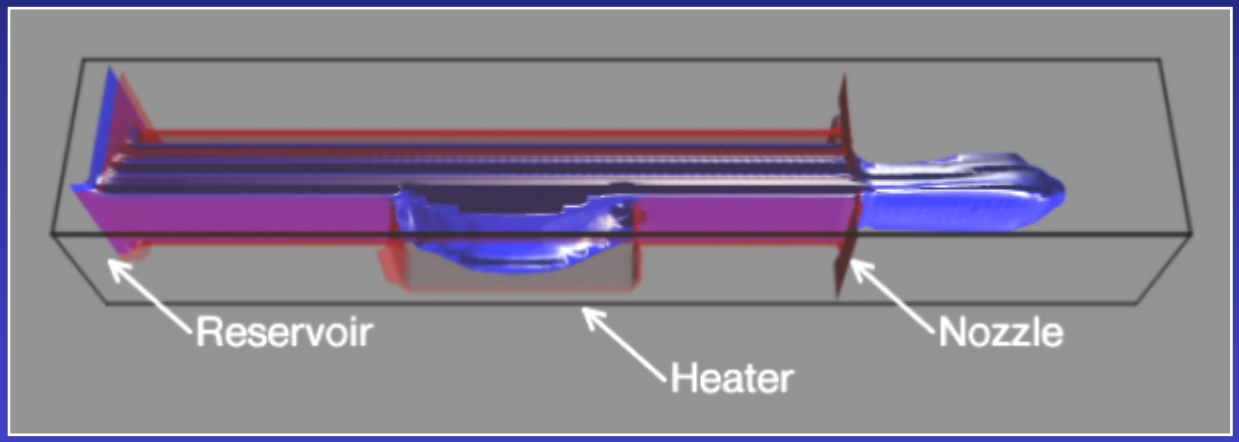


$f(\mathrm{p}) = d(S, \mathrm{p}) - c$, where $S = tS_1 + (1 - t)S_2$

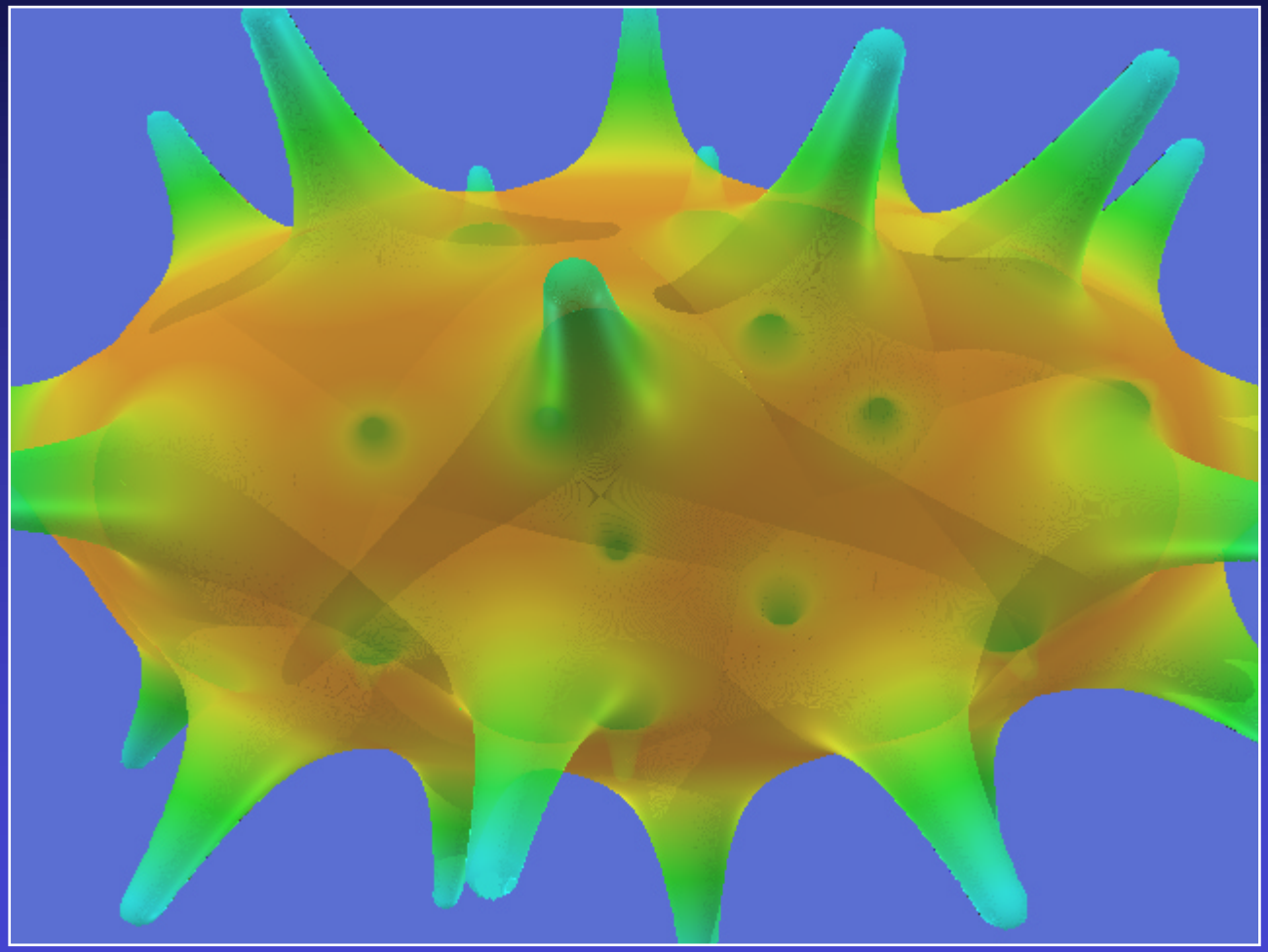


$f(\mathrm{p}) = d(S, \mathrm{p}) - c$, where S is a rigid body rotation between S_1 and S

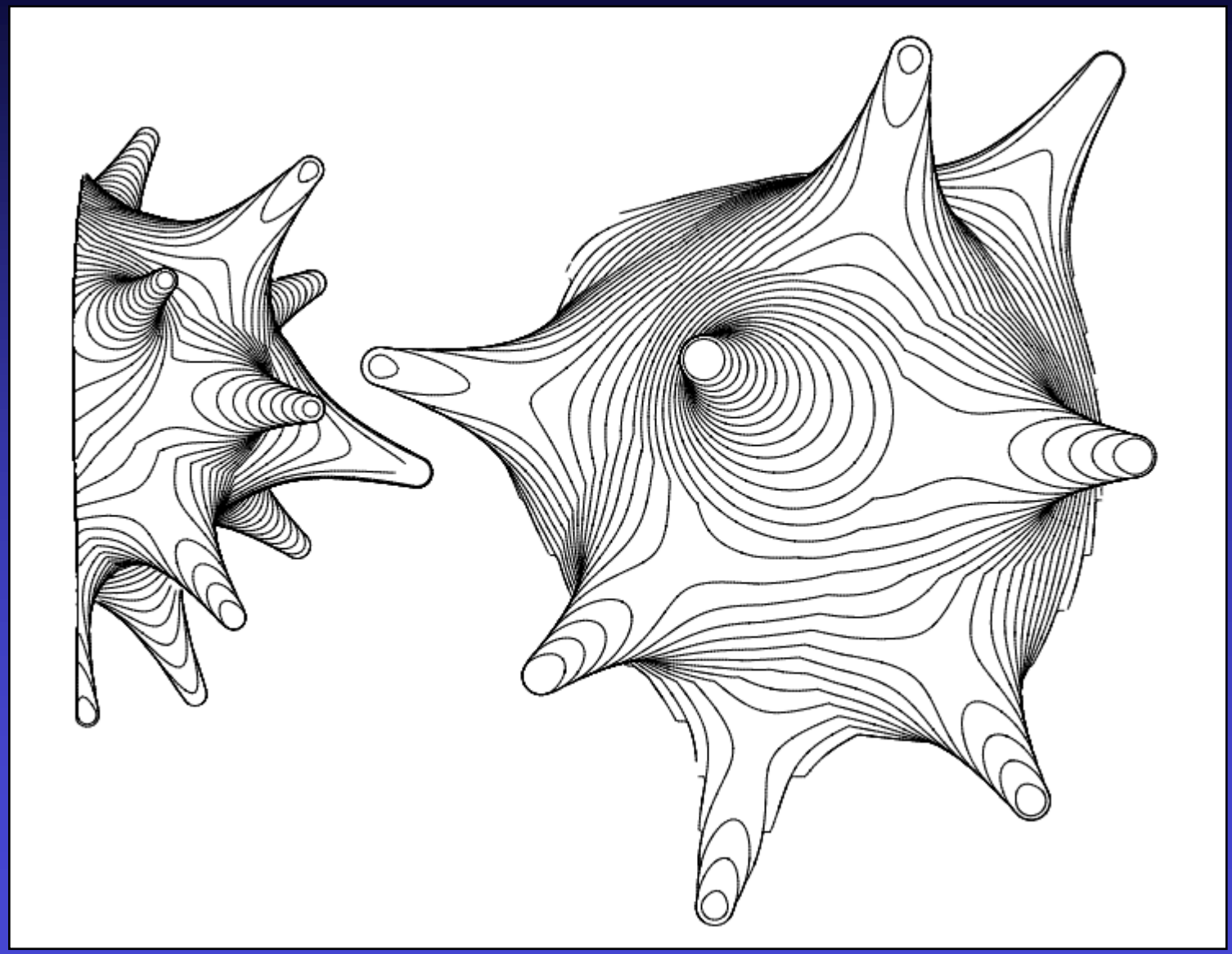
Ink and Printer Head from Discrete Volumetric Data



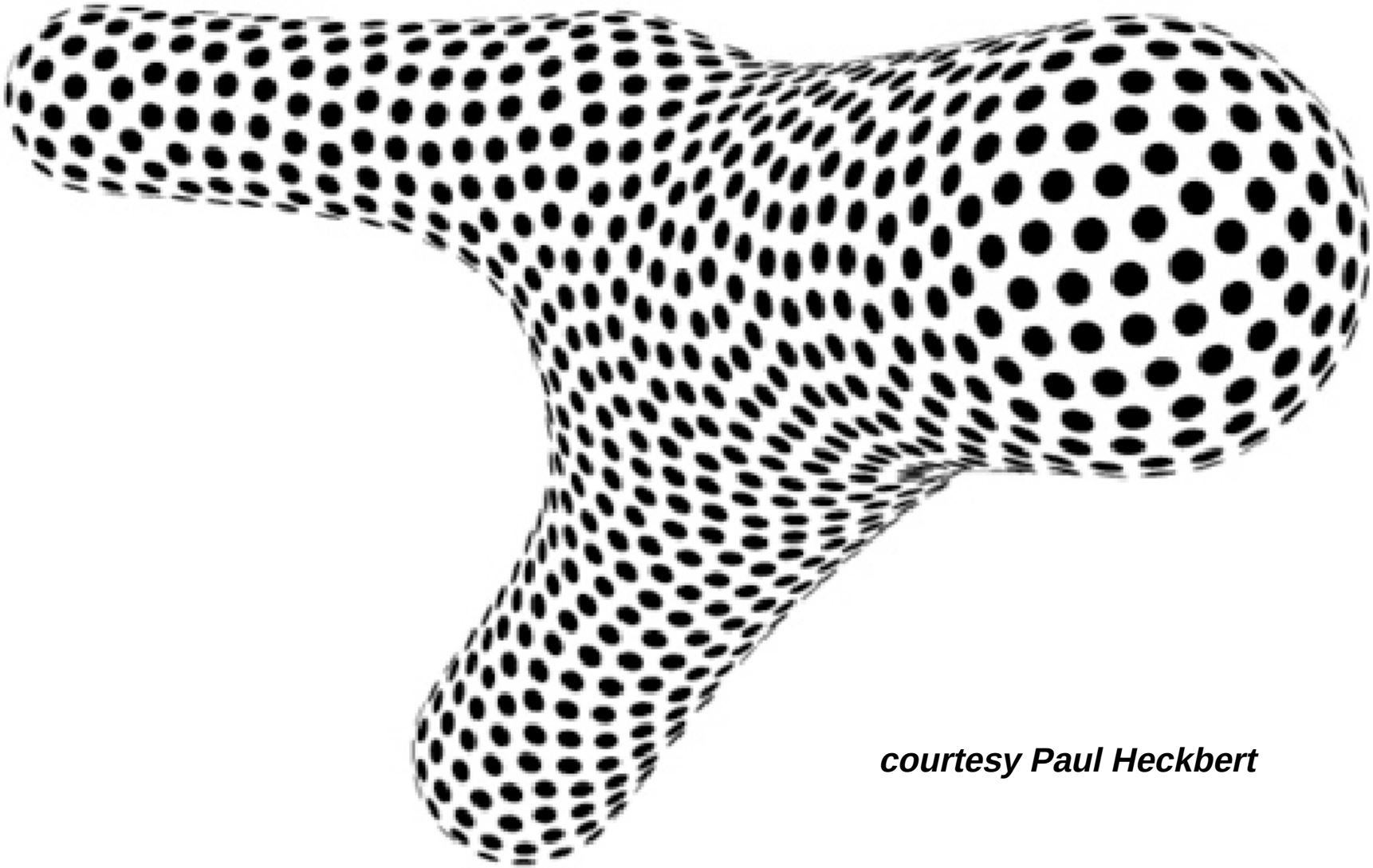
Ray-Traced



Contours Receding from Viewer



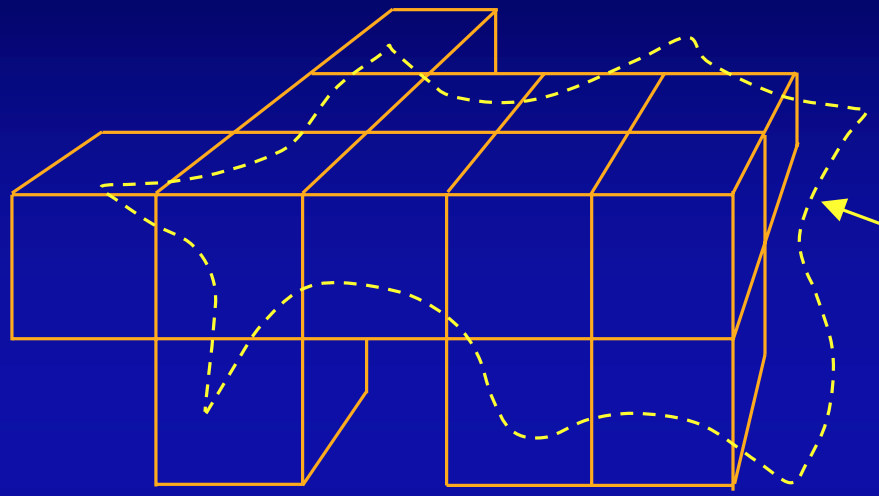
Particle Control and Display

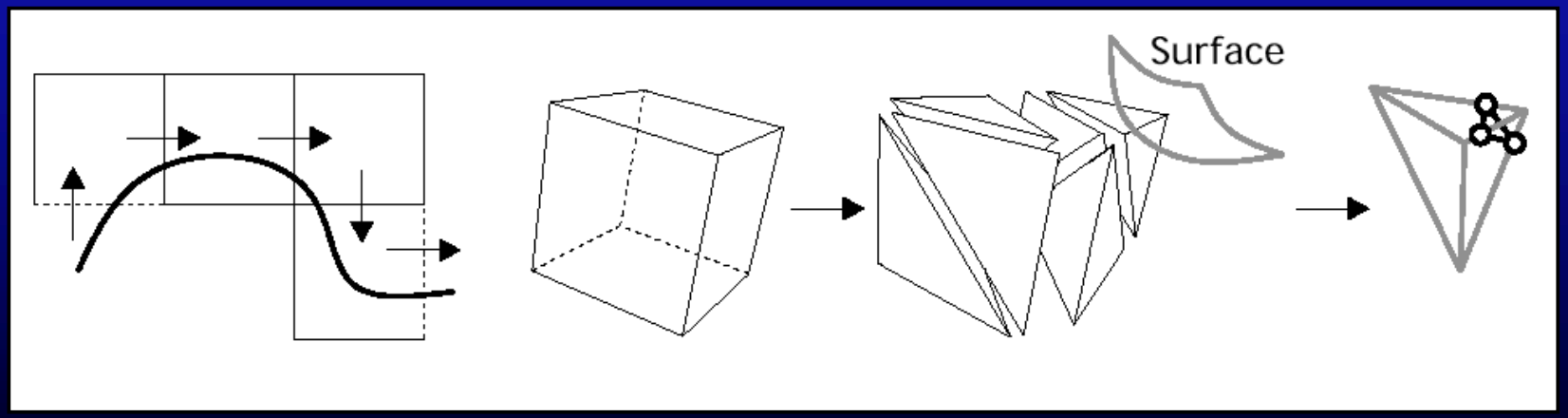


courtesy Paul Heckbert

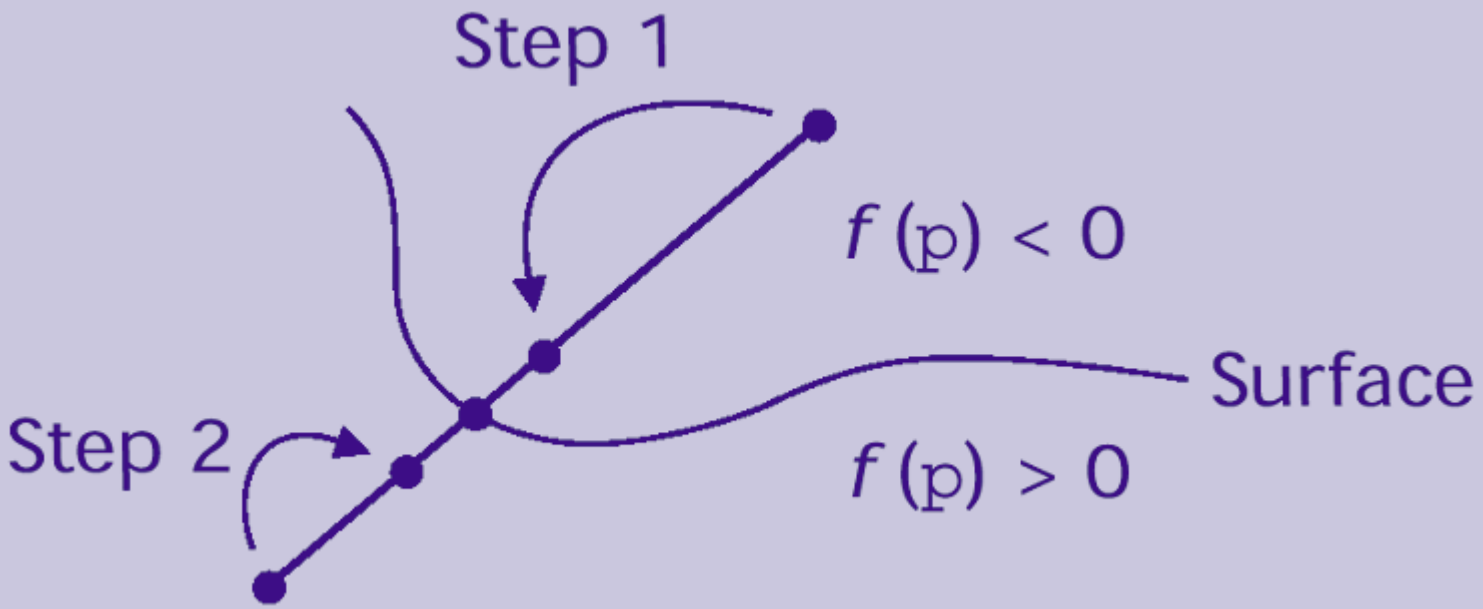
Polygonization by Numerical Continuation

semi-disjoint cells partition space, enclosing object's surface

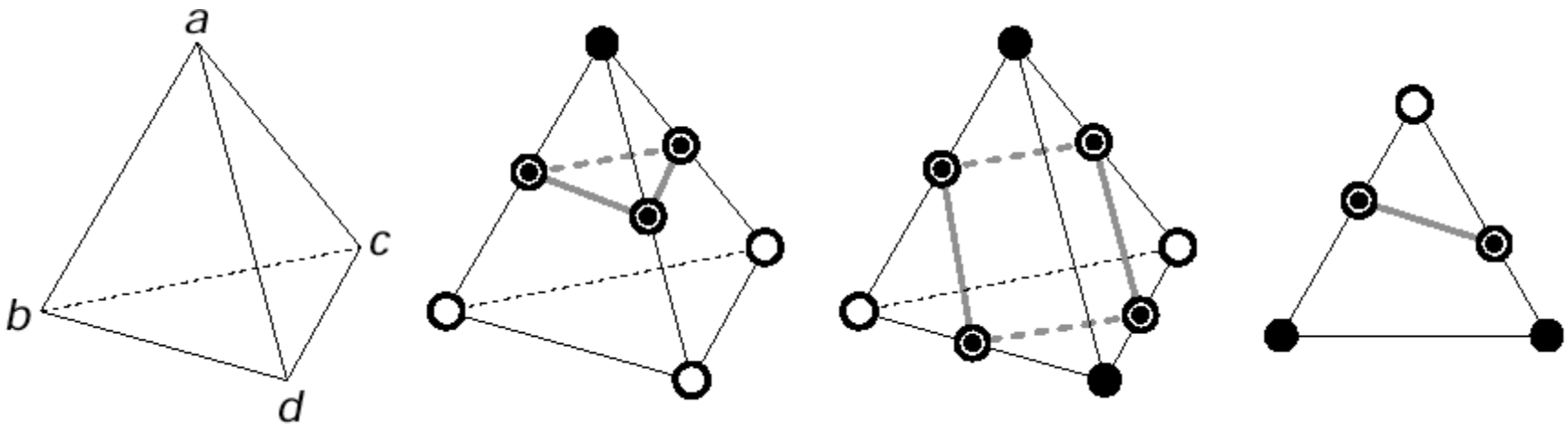




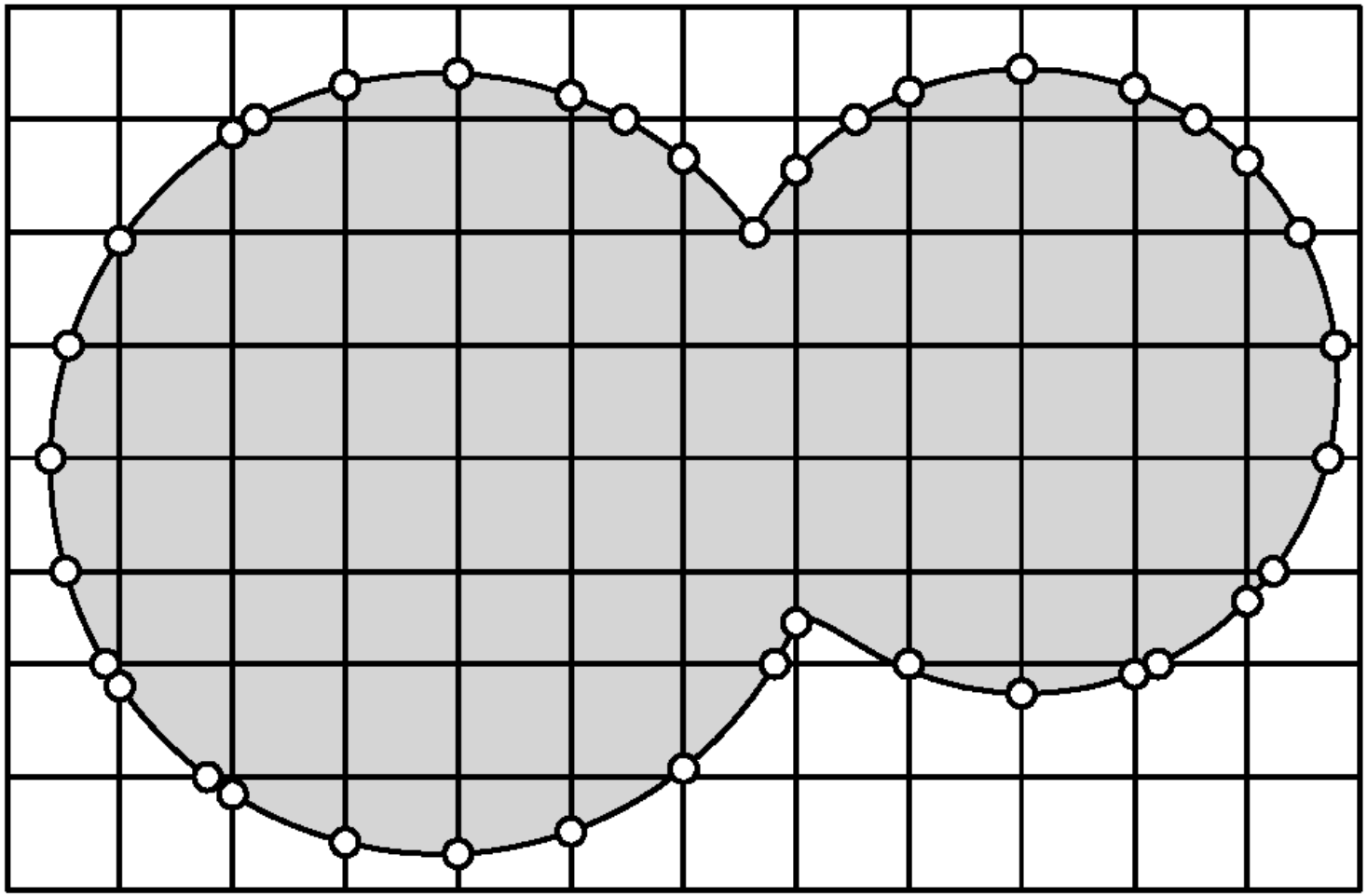
Binary Subdivision for Precise Vertex Location



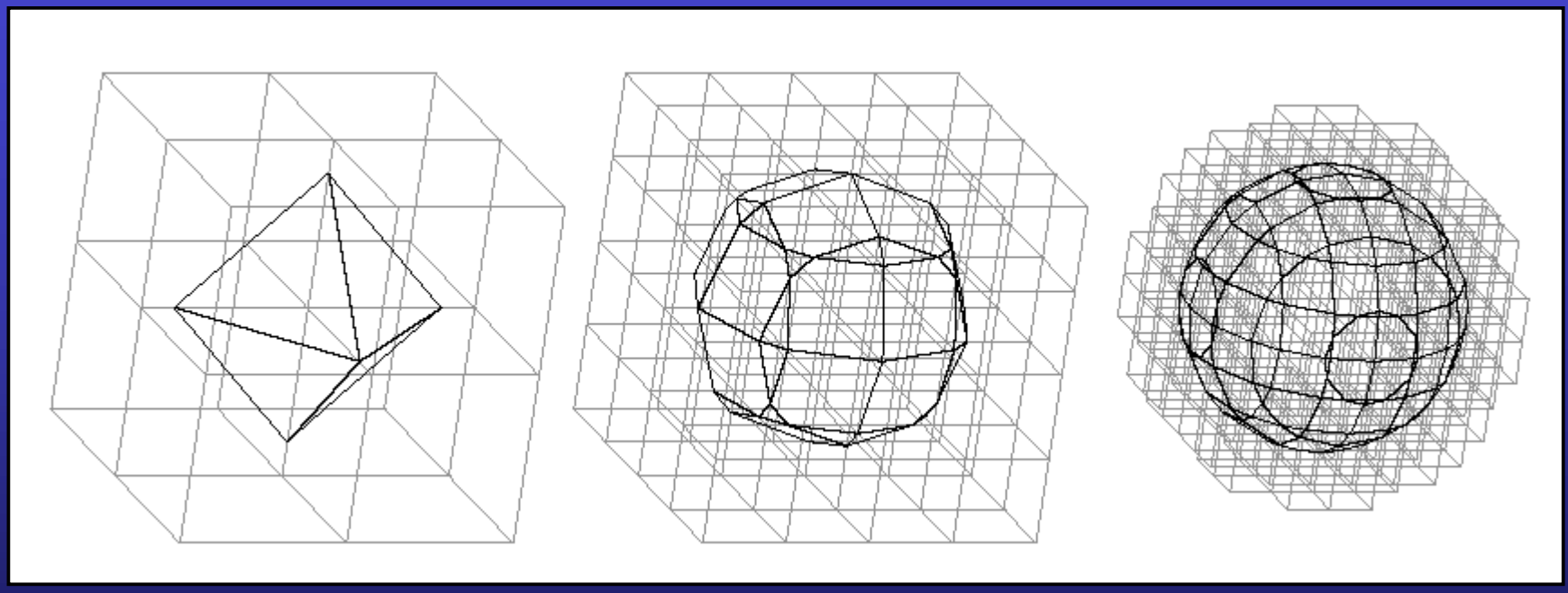
Surface / Tetrahedron Intersections



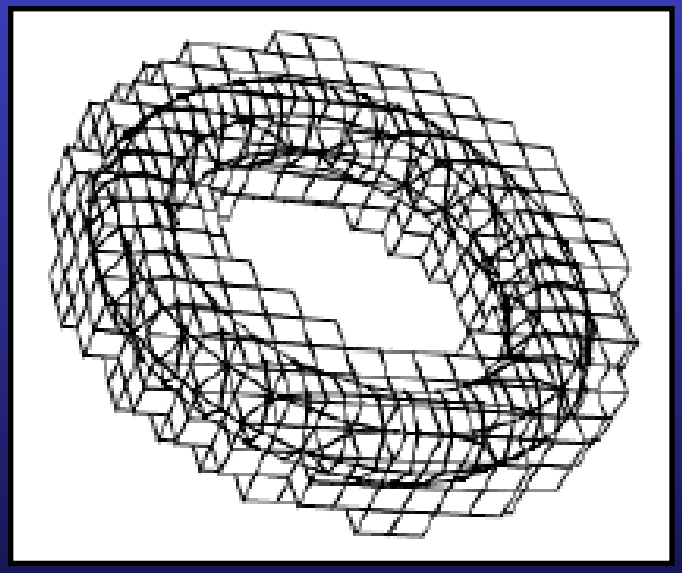
Grid-Based Contour Computation

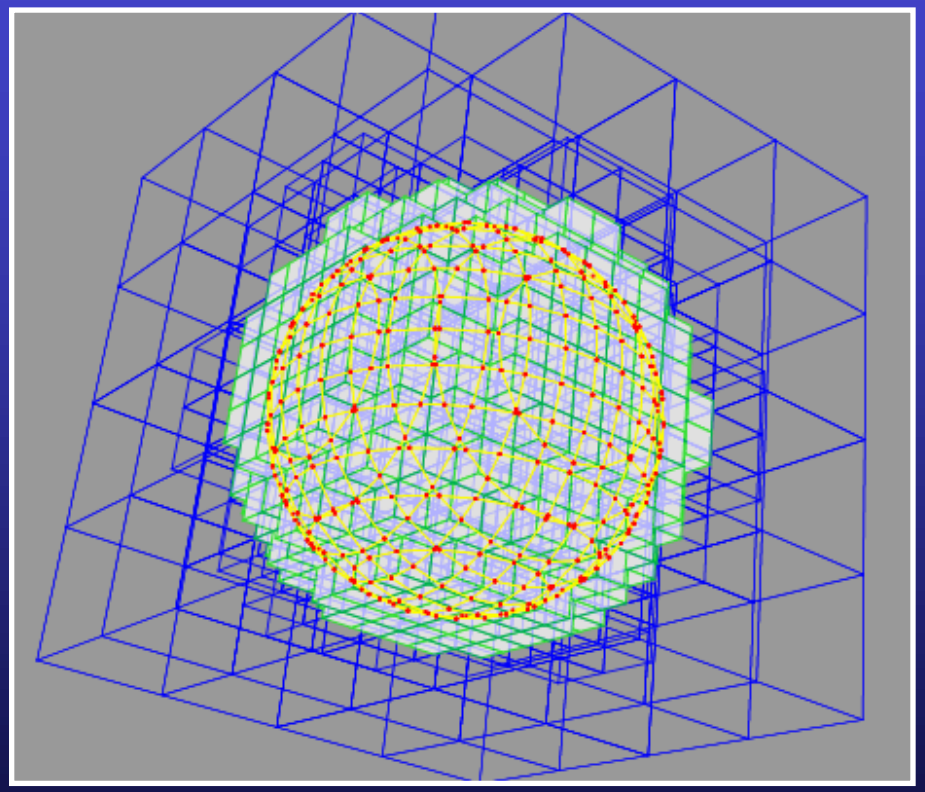


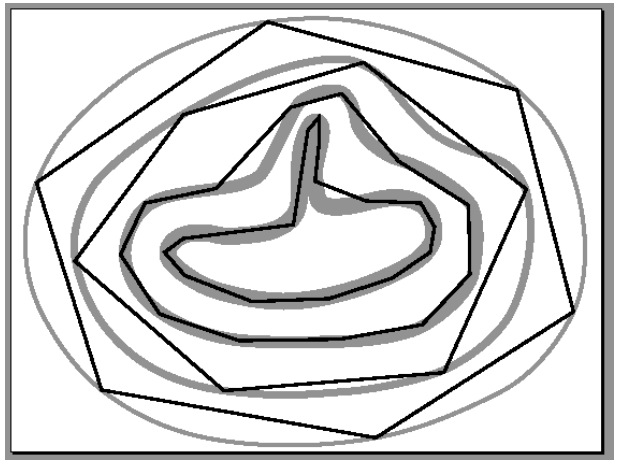
Polygonization Resolution



Polygonized Torus, Sphere

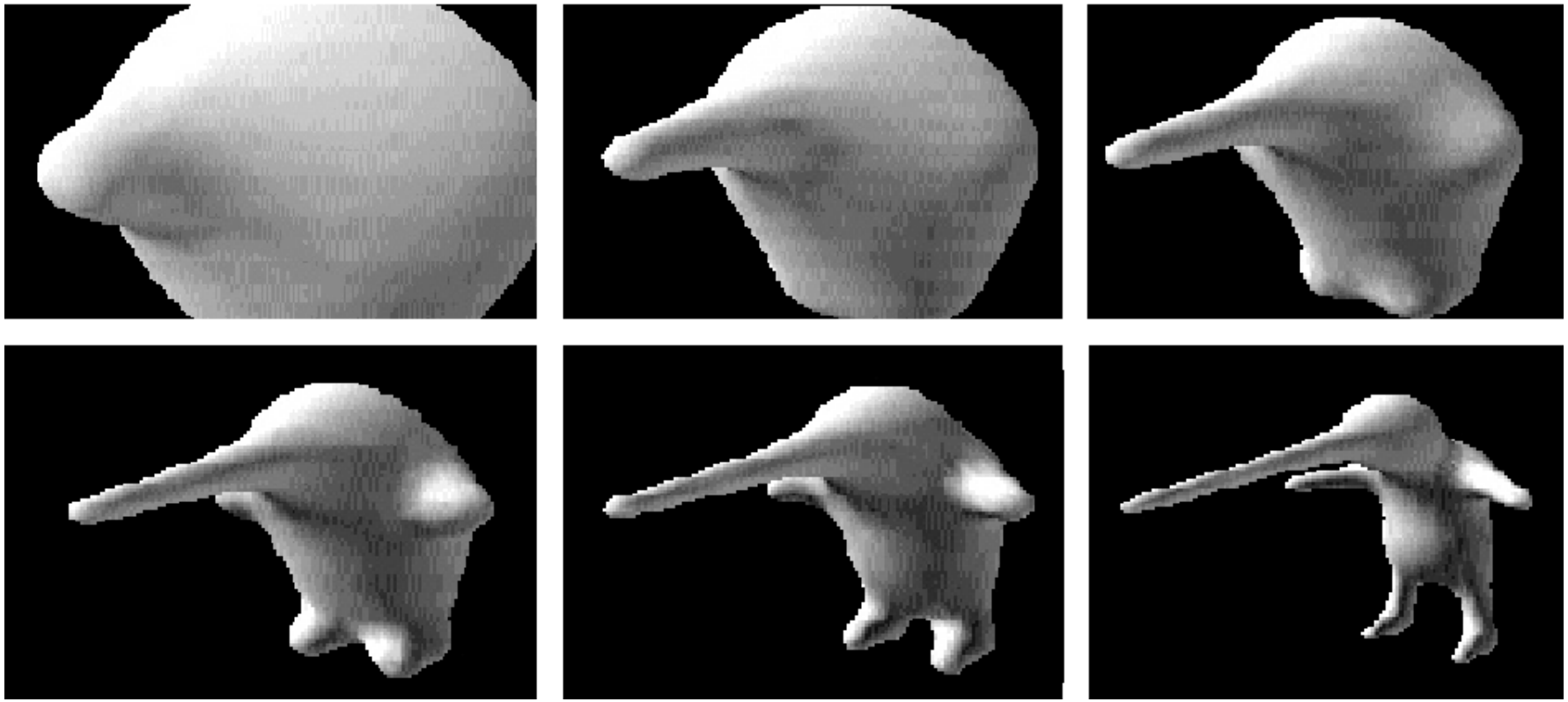




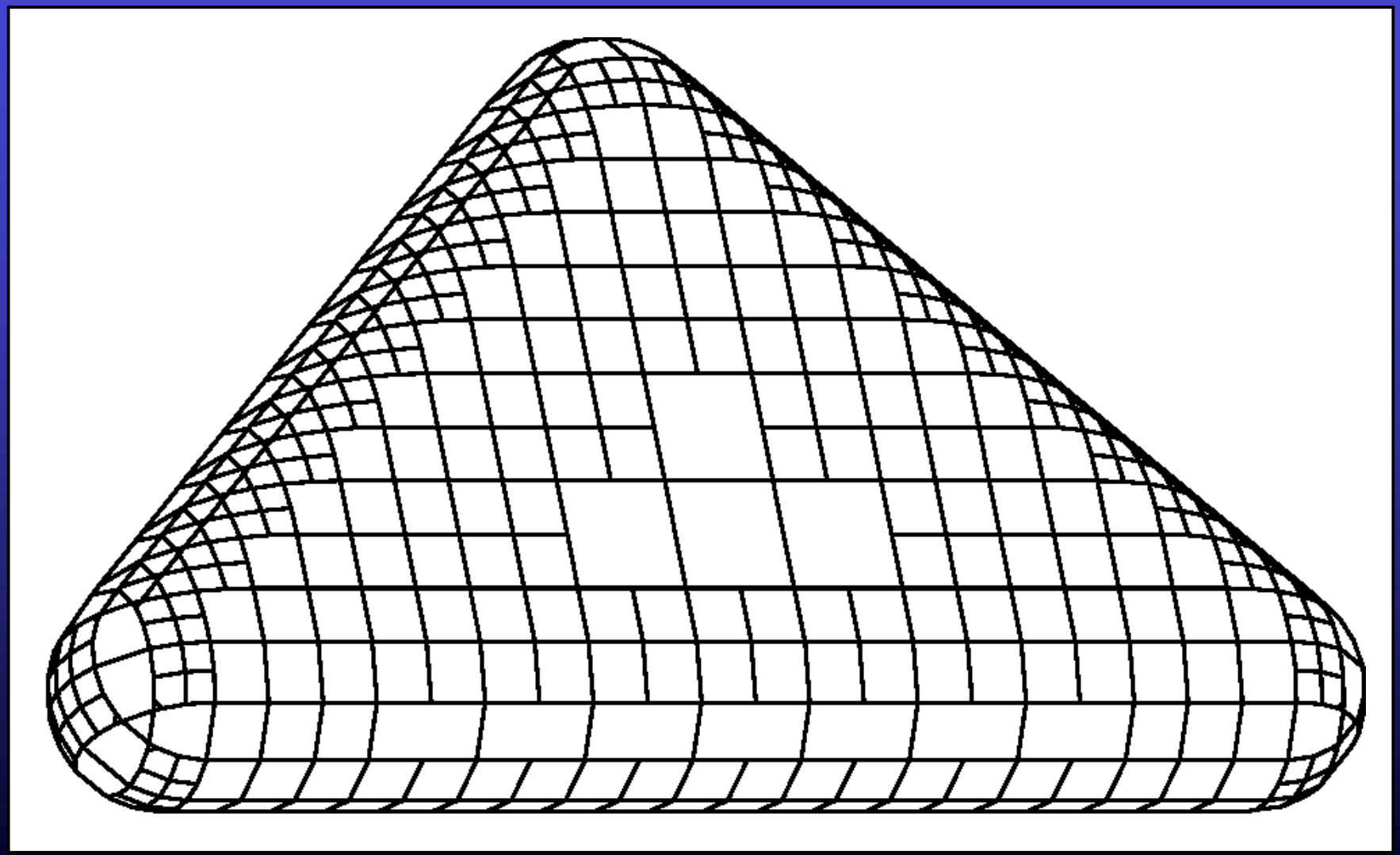


'Shrink-wrap' Polygonization

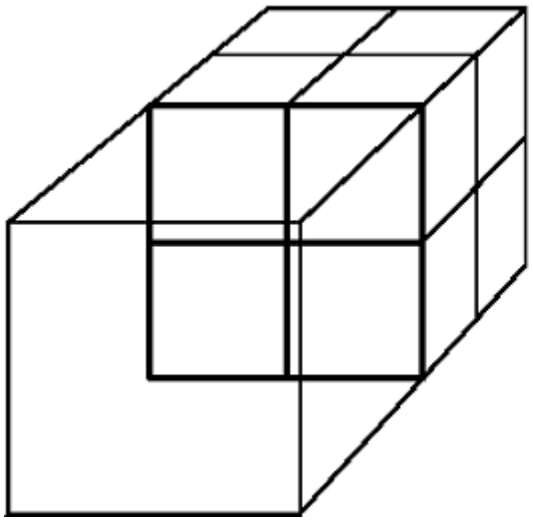
courtesy Brian Wyvill

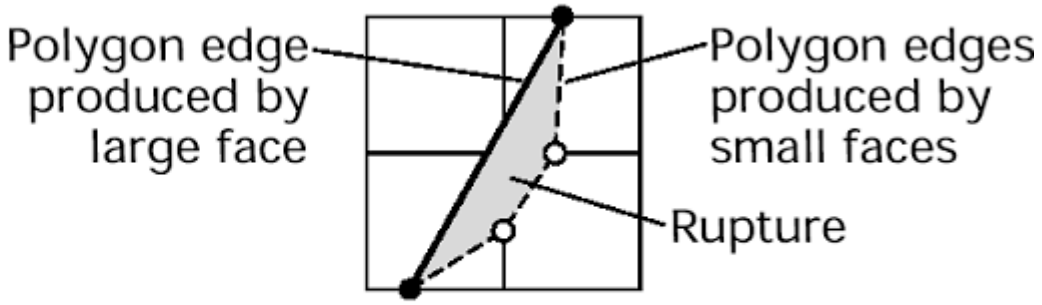


Adaptive Polygonization

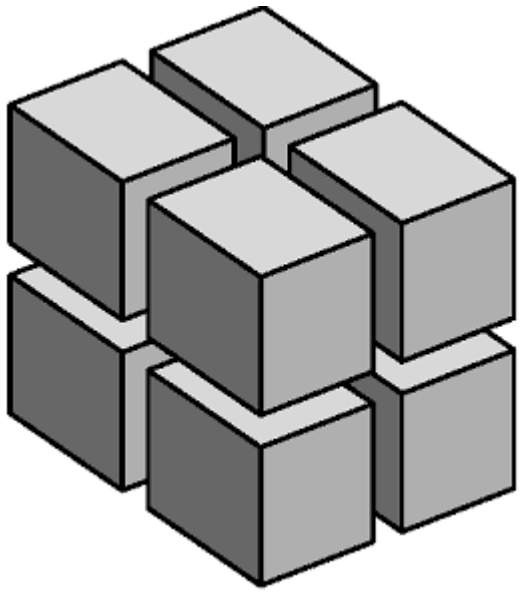


Adaptive Polygonization and 'Cracks'

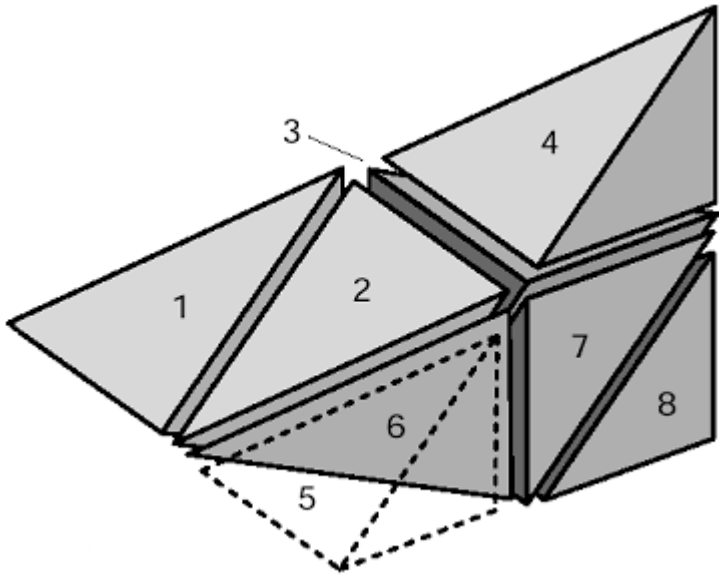




Recursive Subdivision of Cube, Tetrahedron

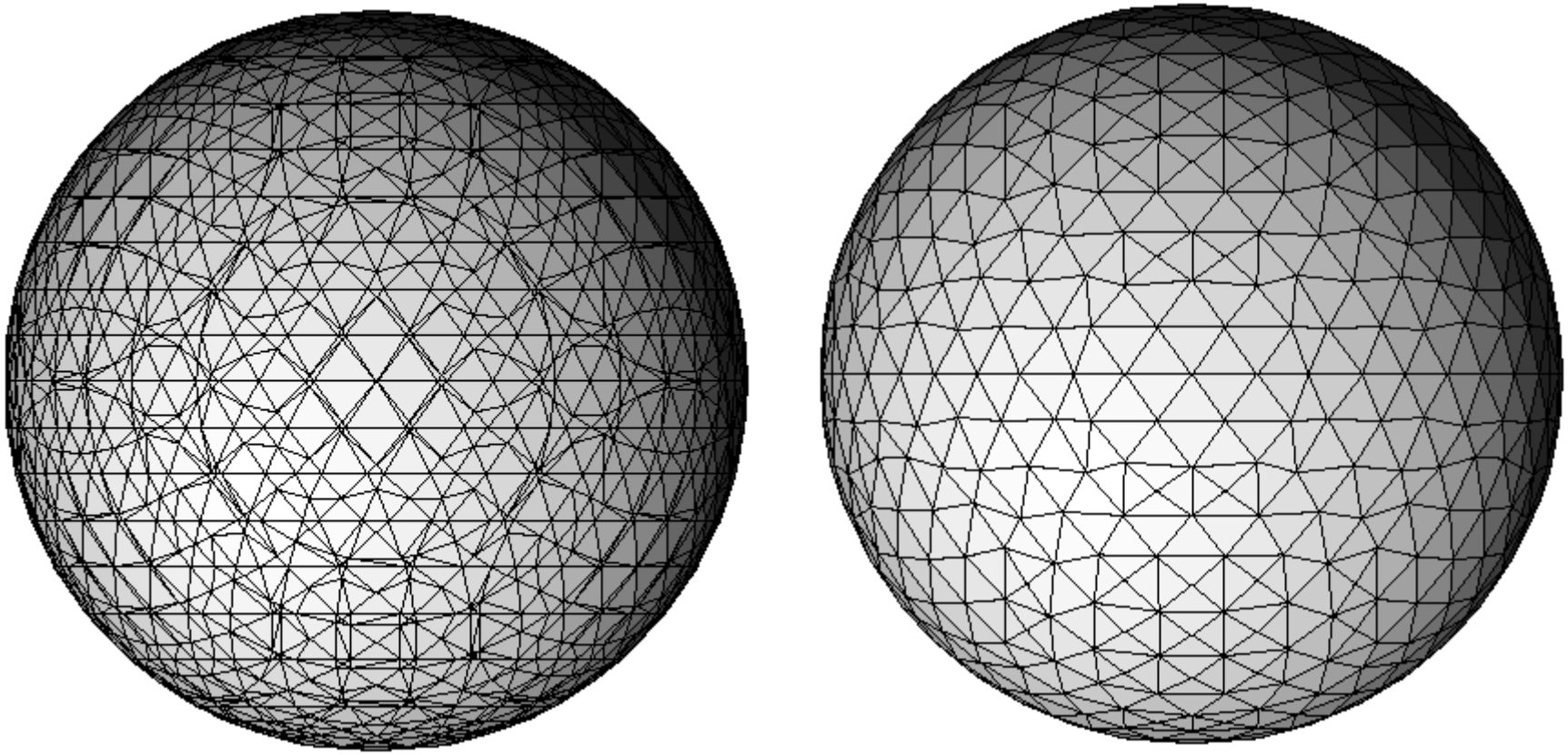


cracks possible with cubic subdivision



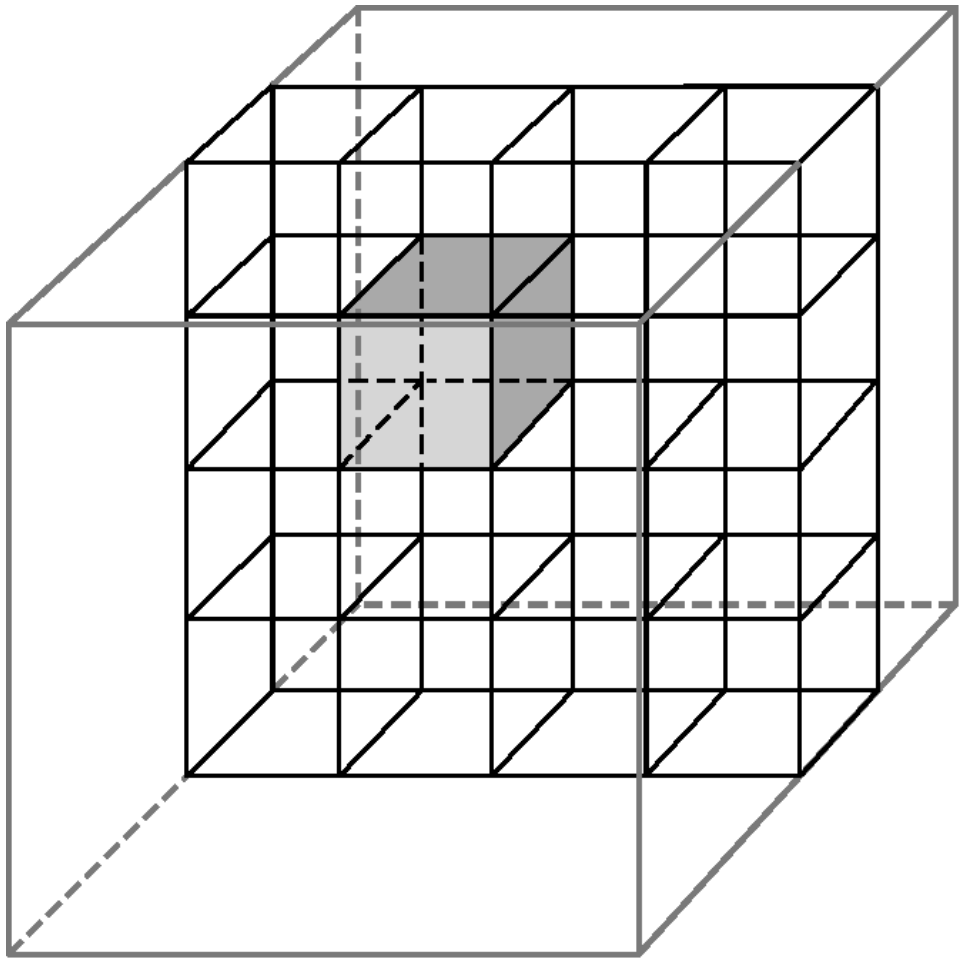
***Kuhn simplex* maintains crack-free 'honeycomb'**

Triangle Optimization during Polygonization

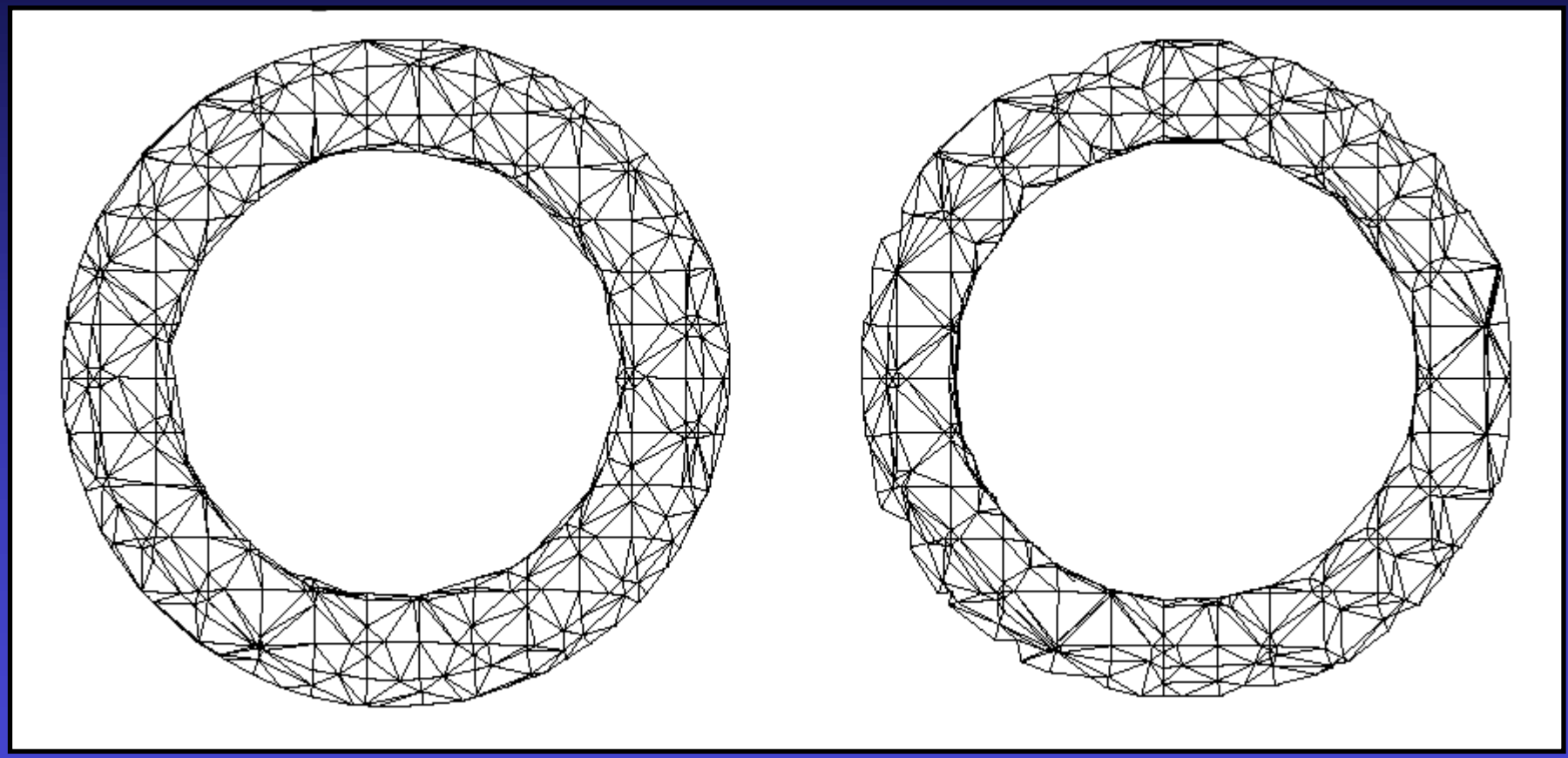


courtesy
Doug Moore

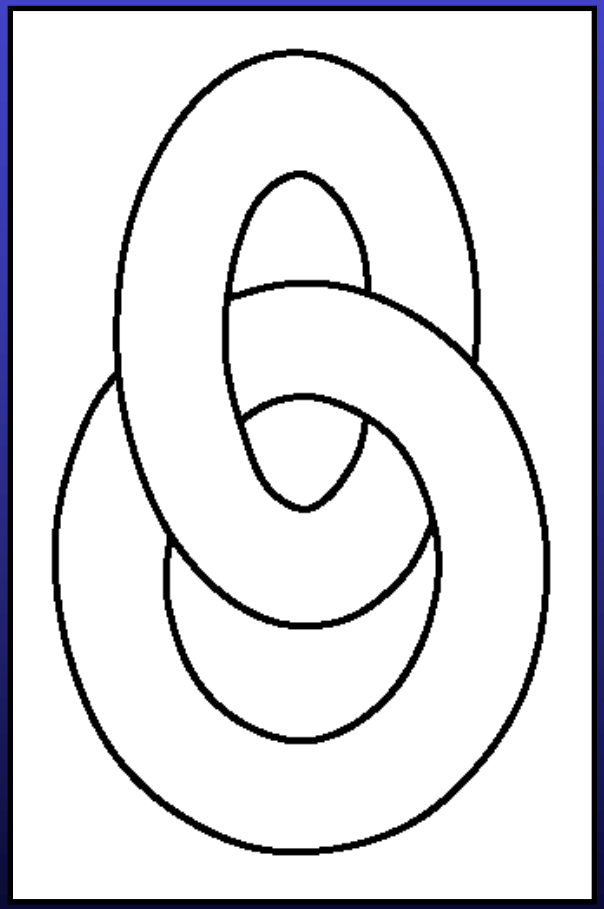
Marching Cubes

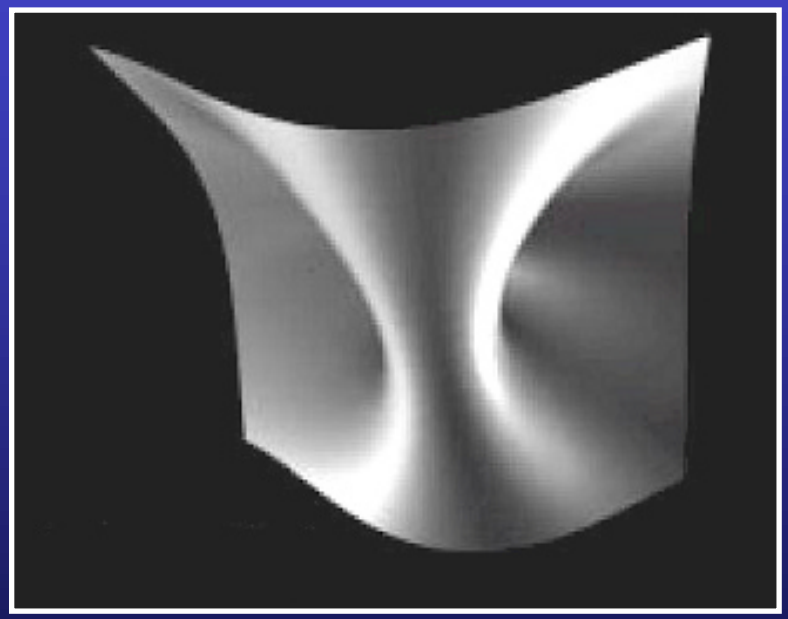


Binary Subdivision and Linear Interpolation



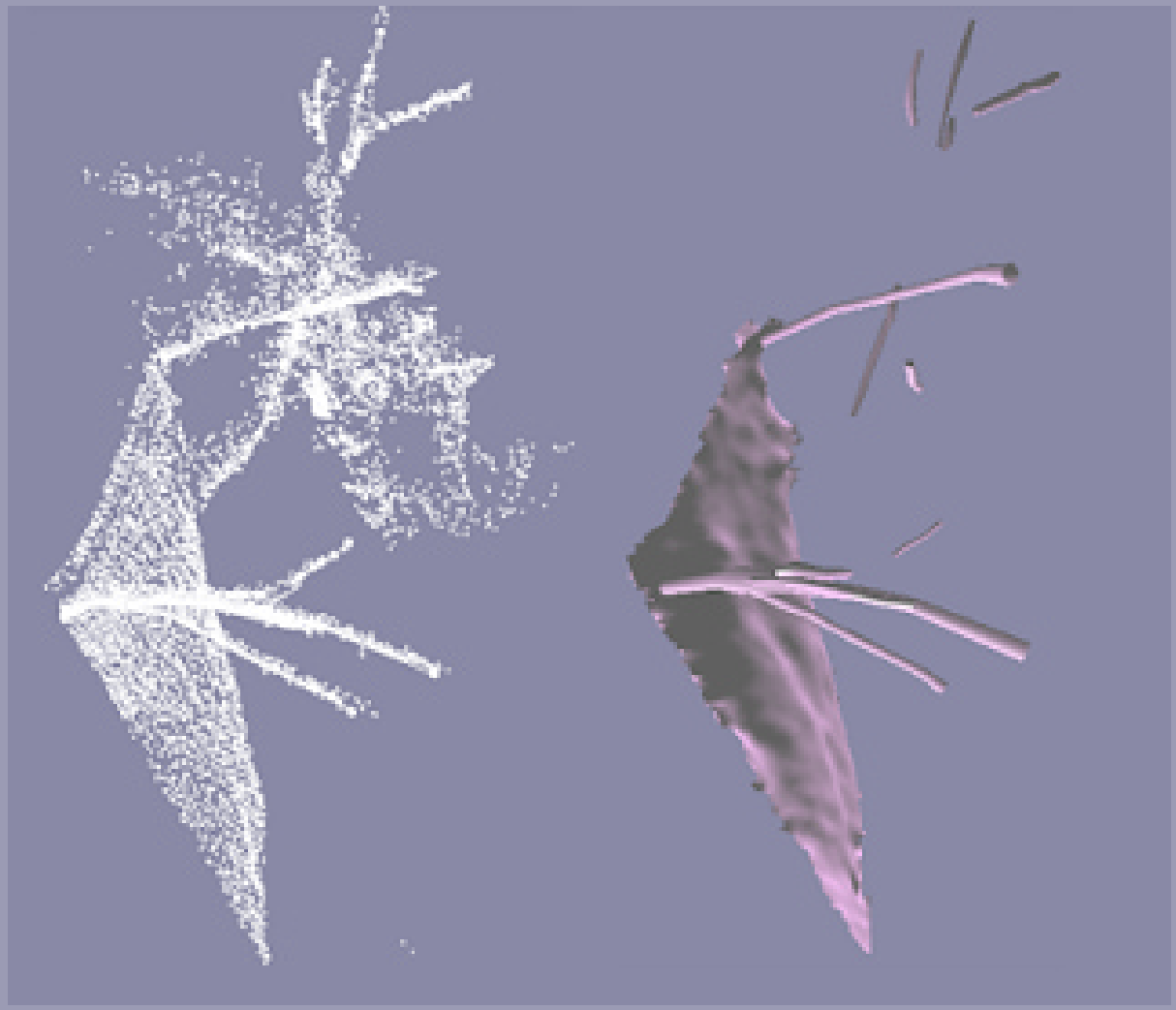
Surface Equi-distant from Two Interlocking Tori

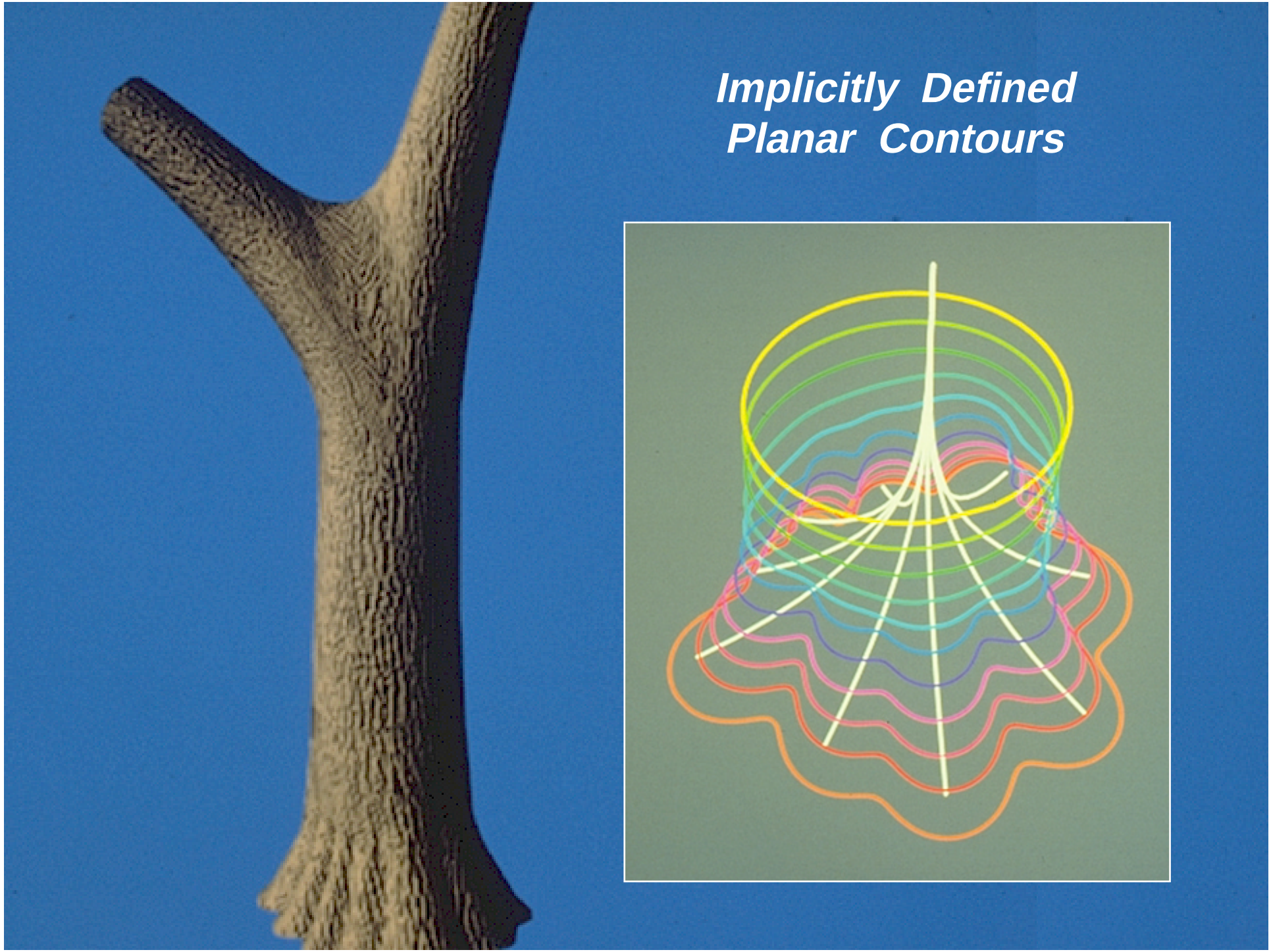




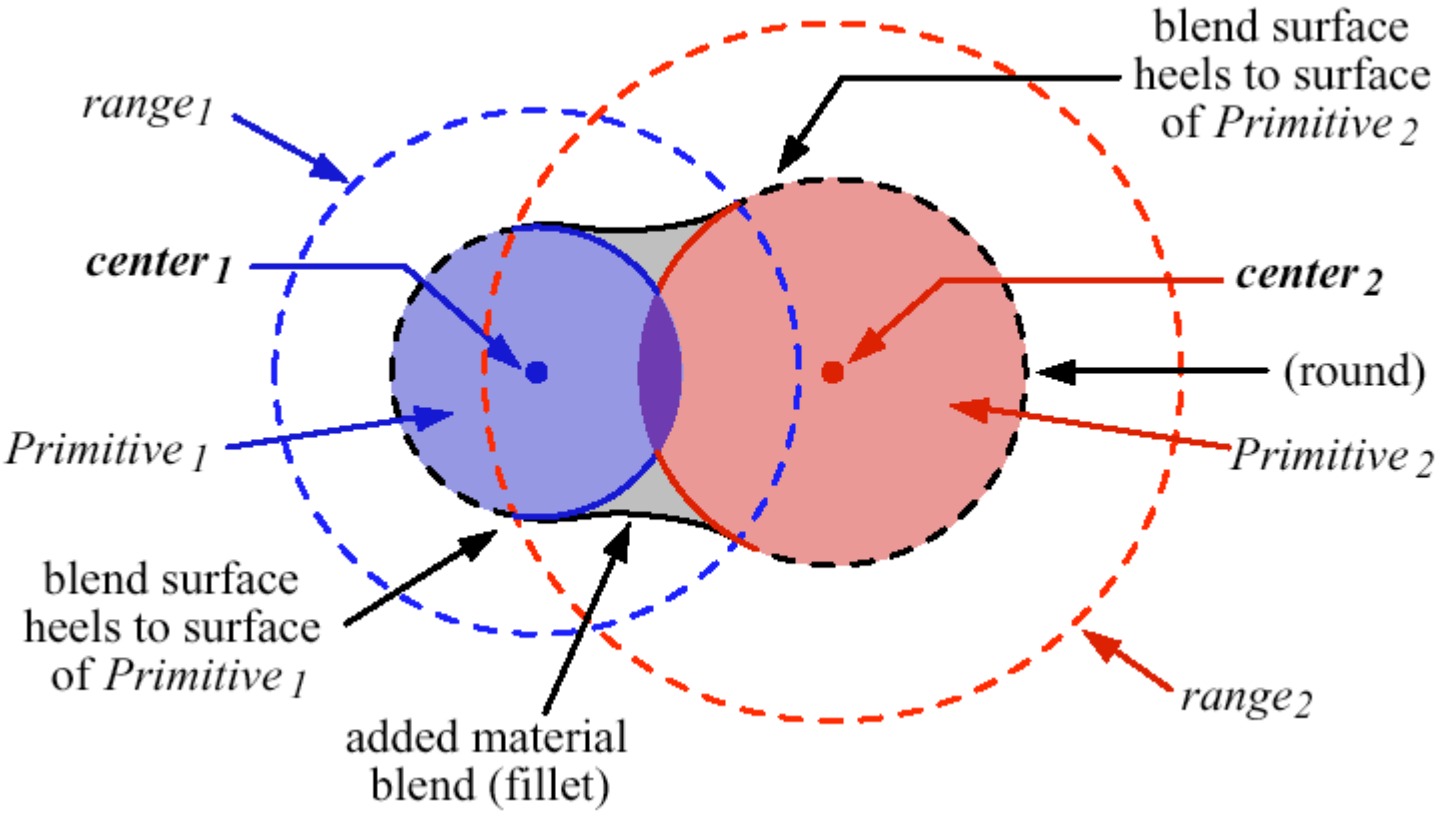
courtesy
Brian Wyvill

Reconstruction from Cloud of Points



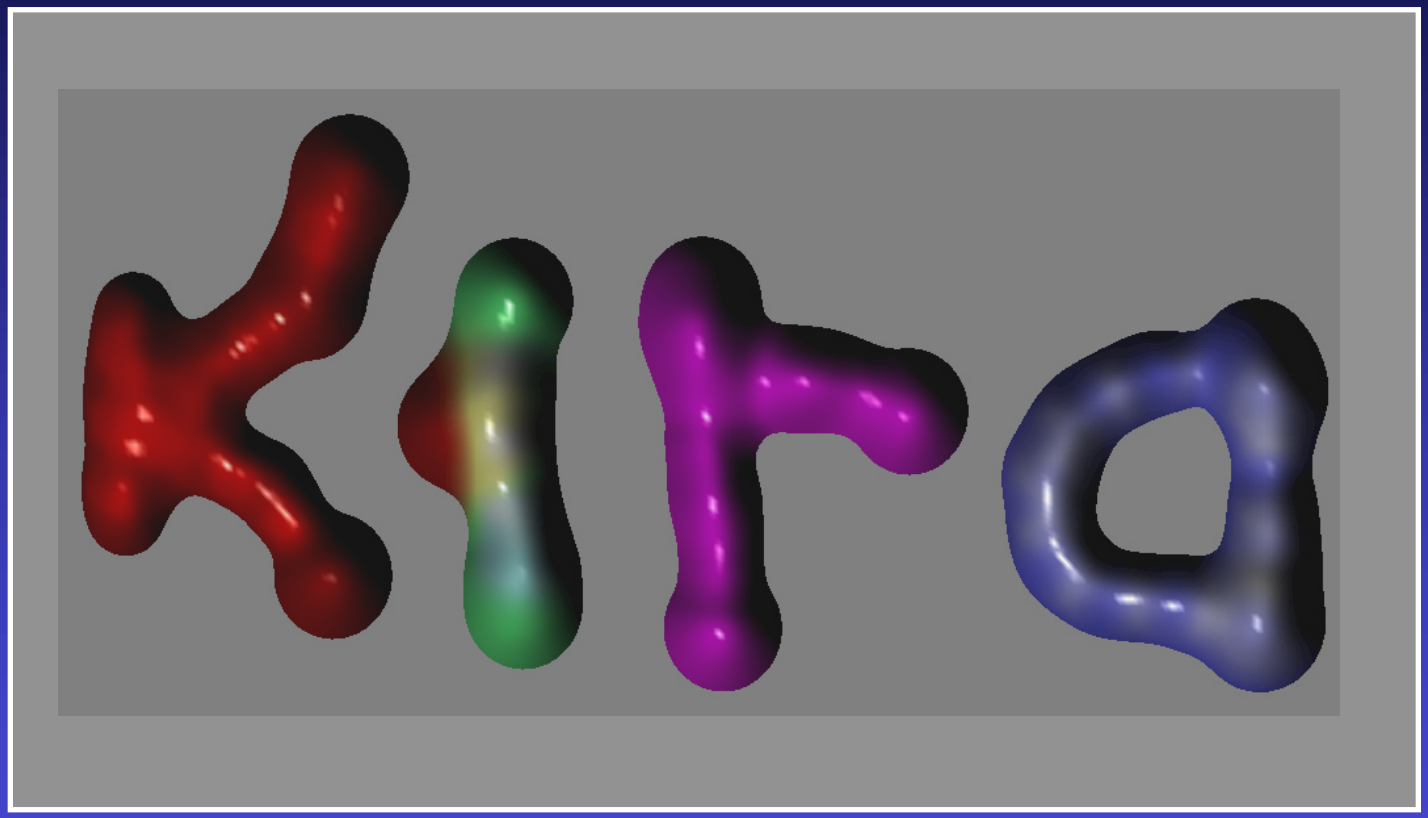

Implicitly Defined
Planar Contours

Blend Terminology



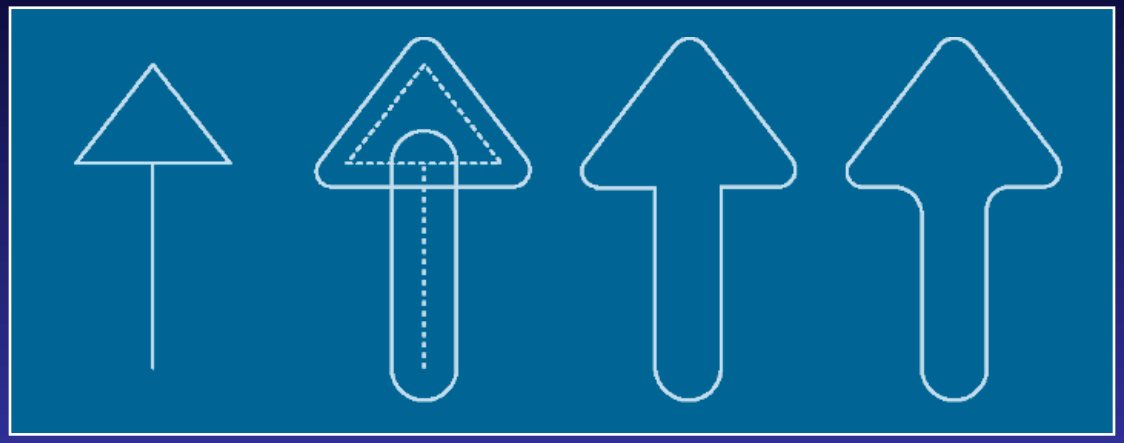
‘Blobby Molecules’

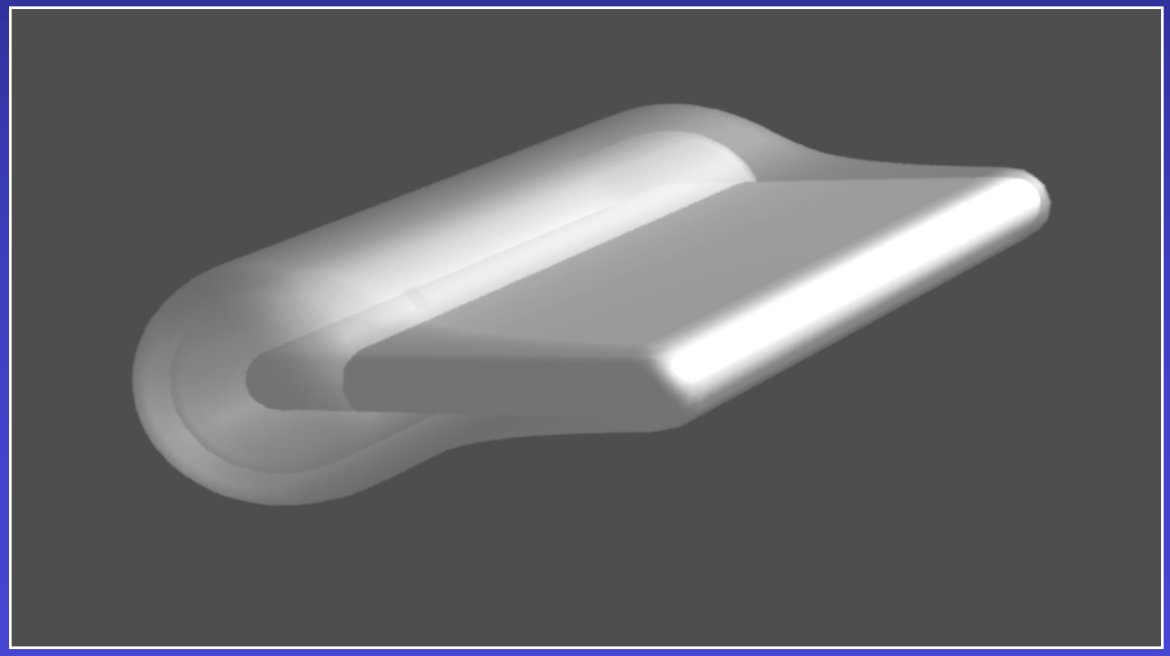
real-time motion and display



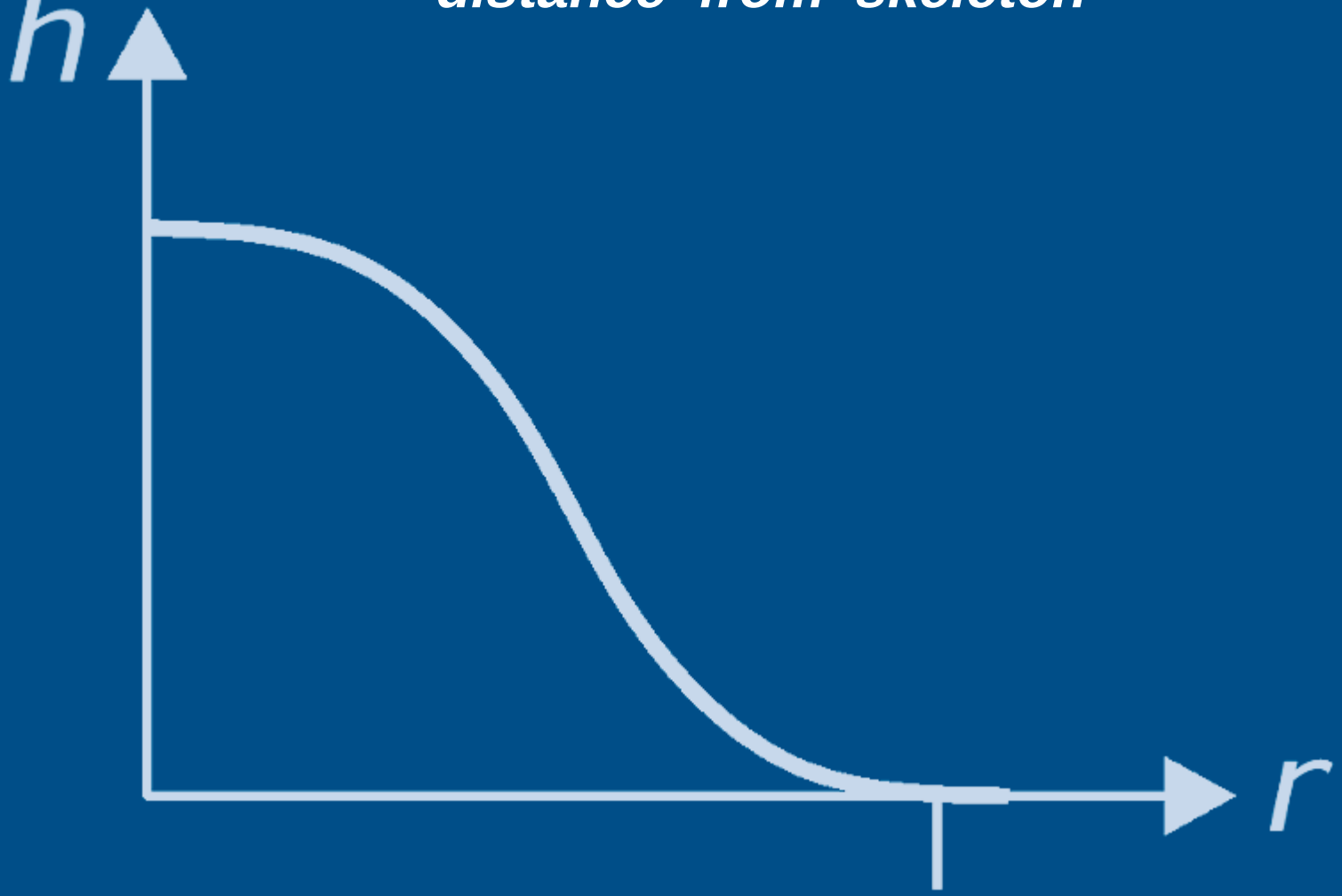
courtesy Brian Wyvill

Skeletal Fields

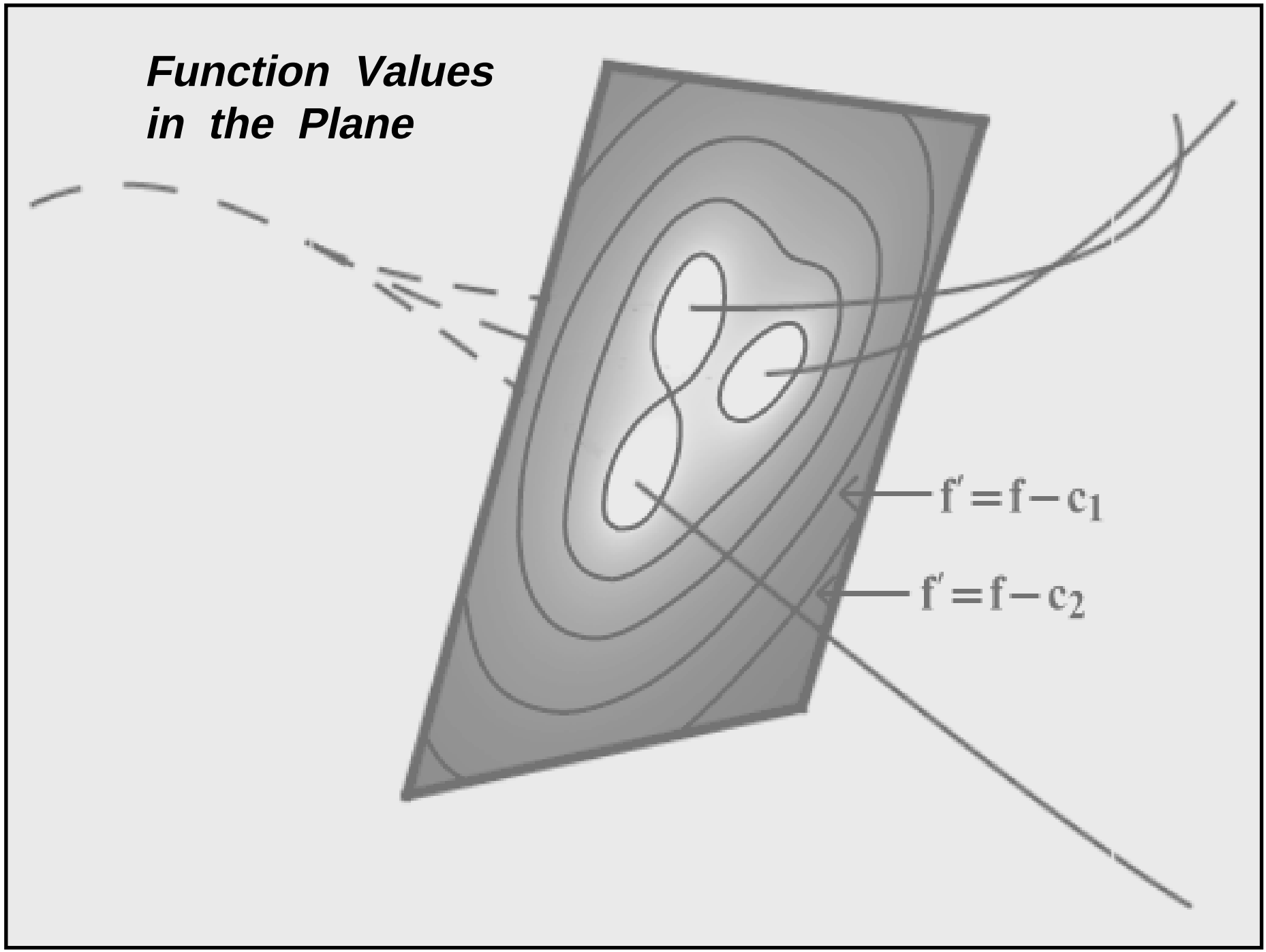


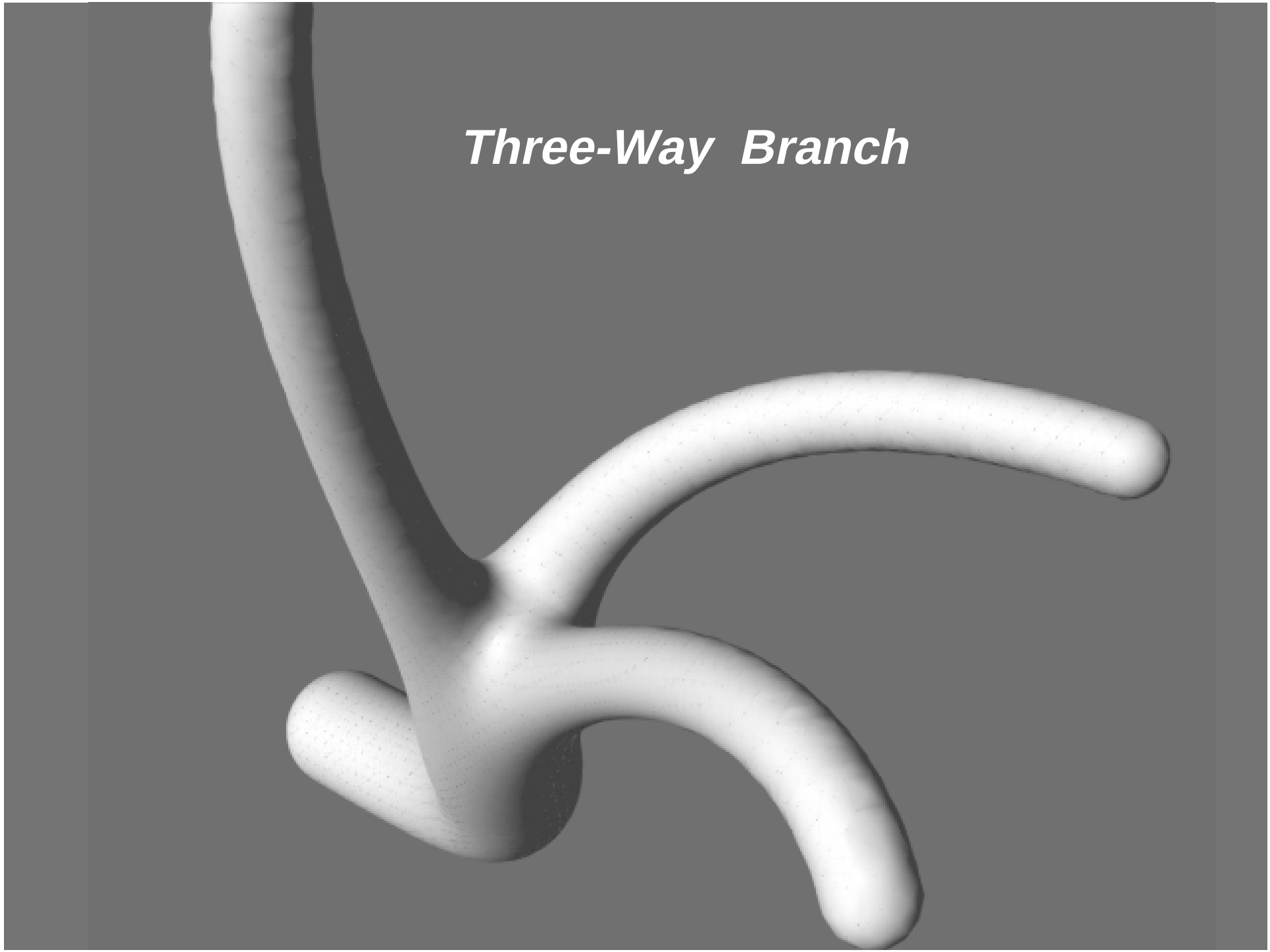


Field h a function of distance from skeleton

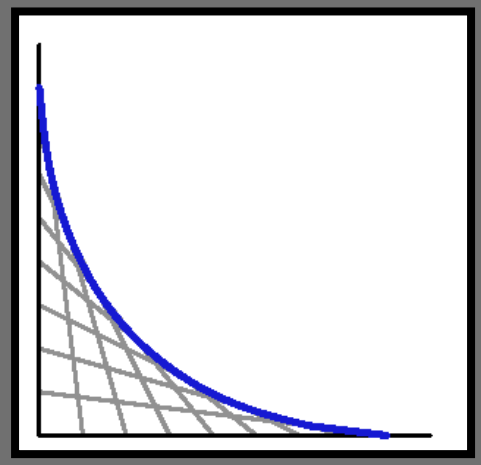


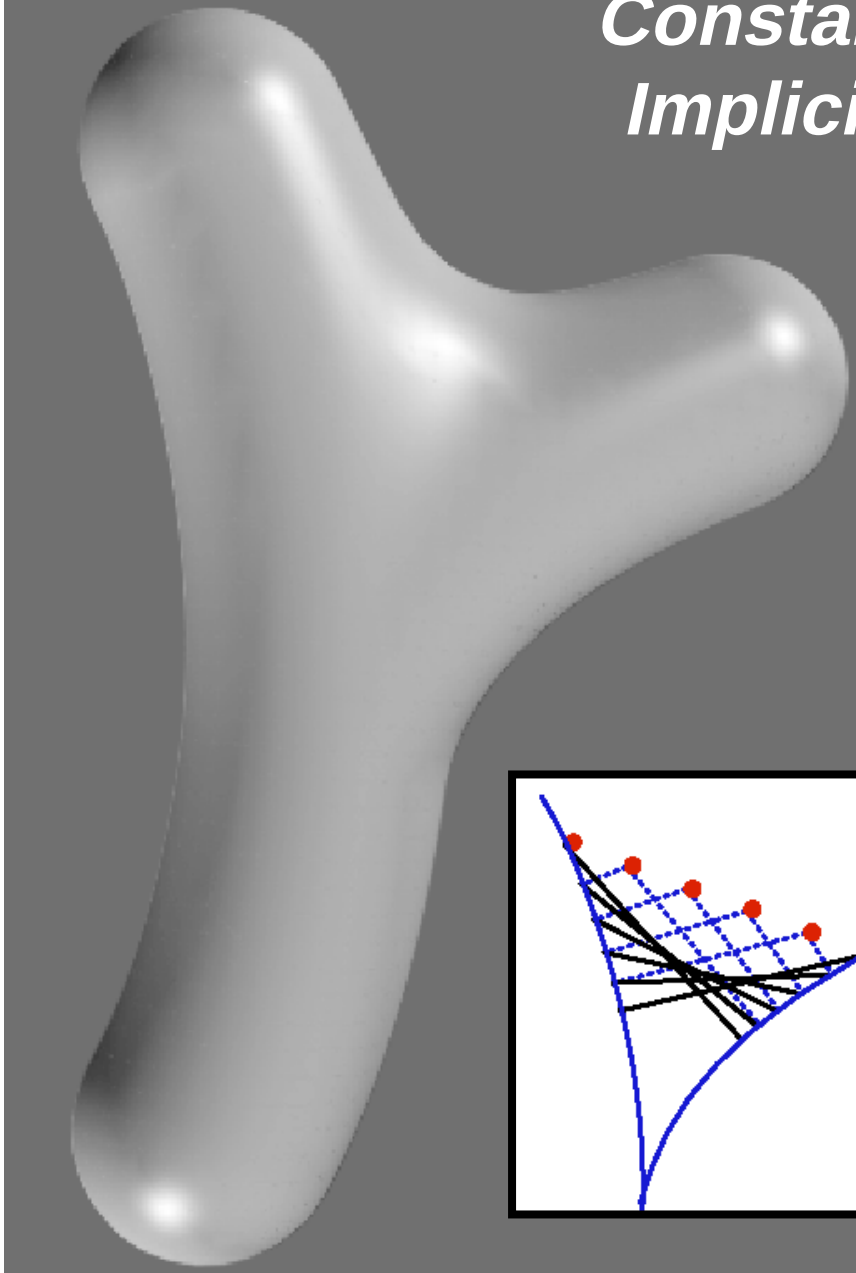
Function Values in the Plane

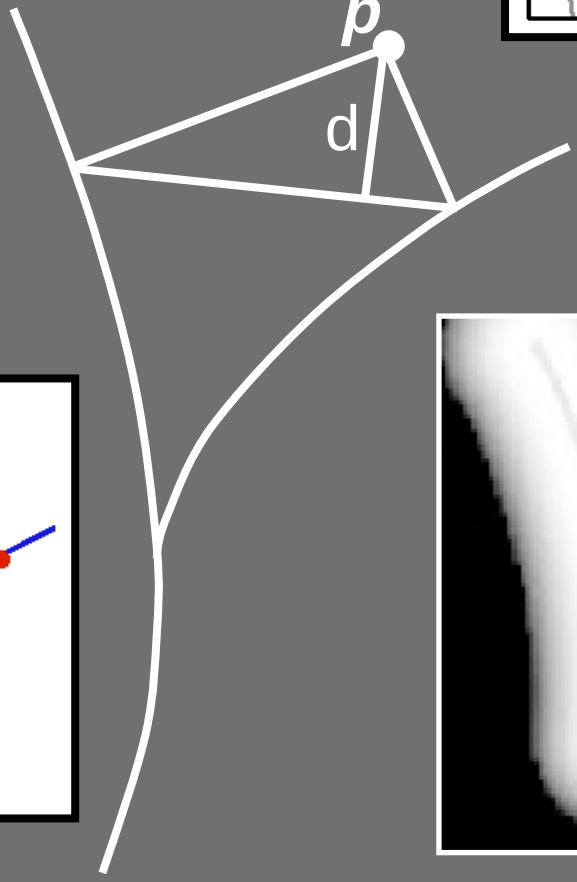


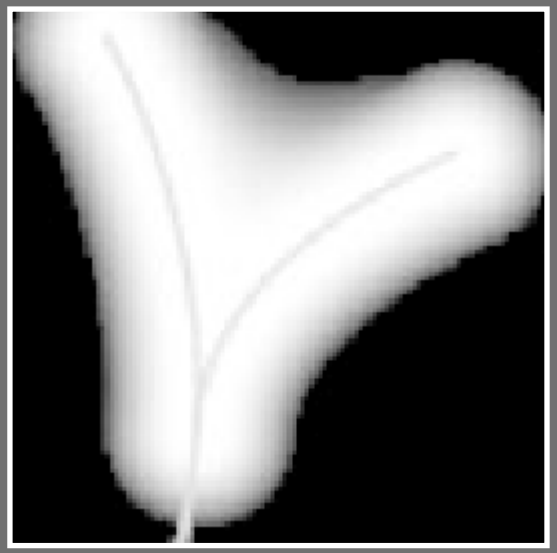

Three-Way Branch

Constant Radius Implicit Branch

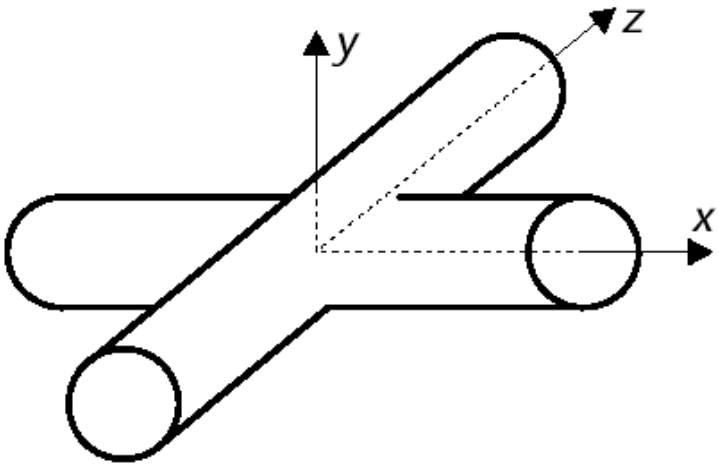


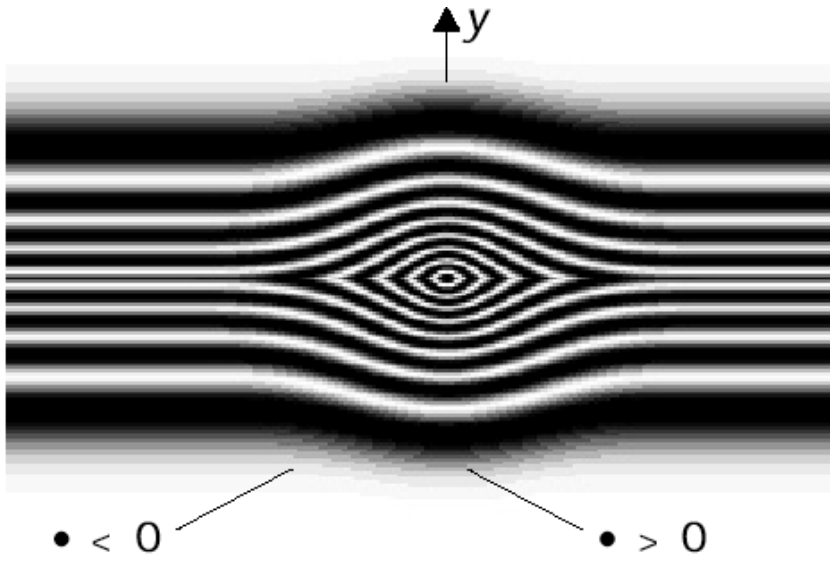






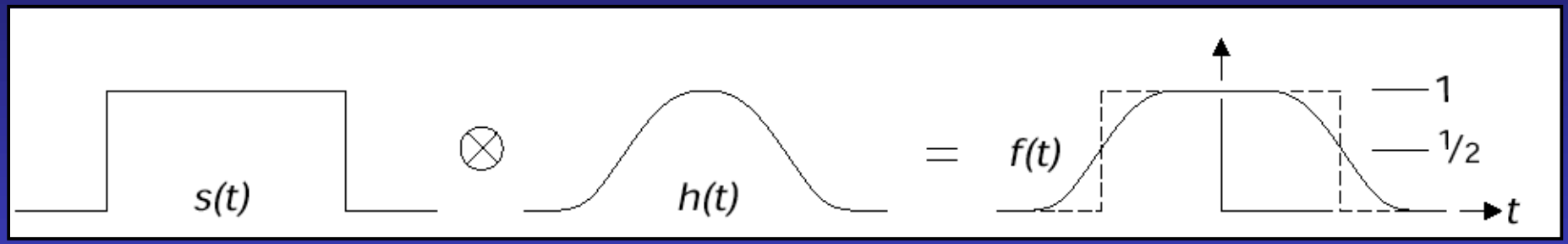
Bulging from Overlapped Cylinders

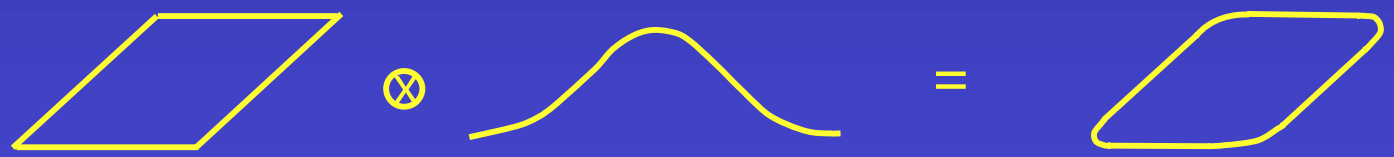




Convolution

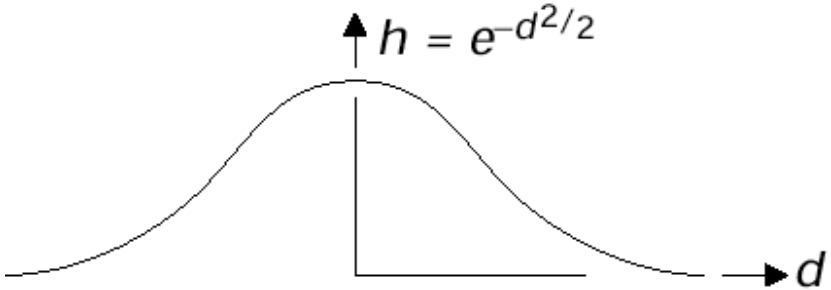
- rounds corners, fillets crevices
- produces a smooth, differentiable field

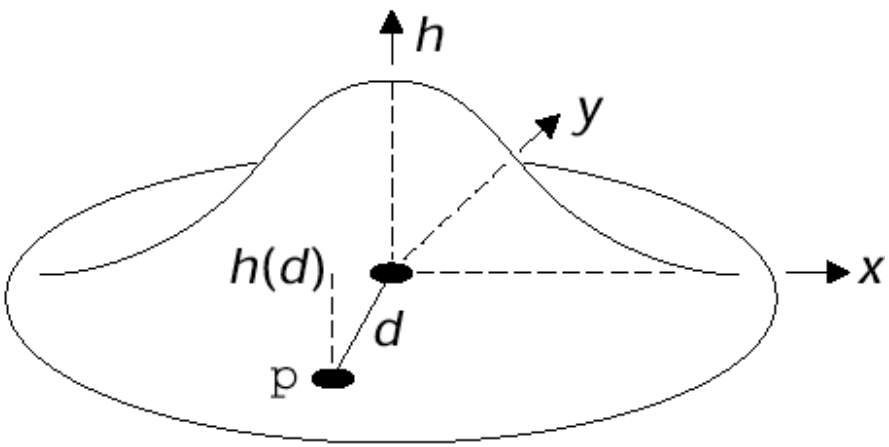




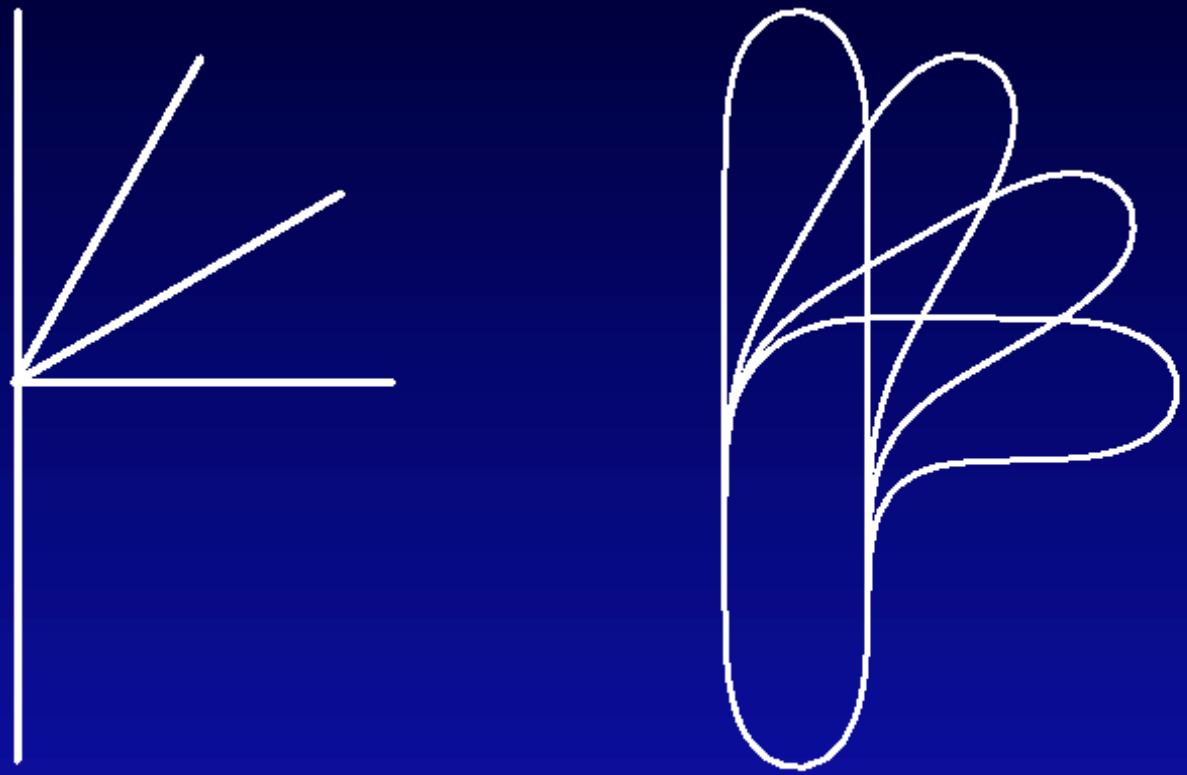
object | filter kernel | smoothed object

1D and 2D Convolution Kernels

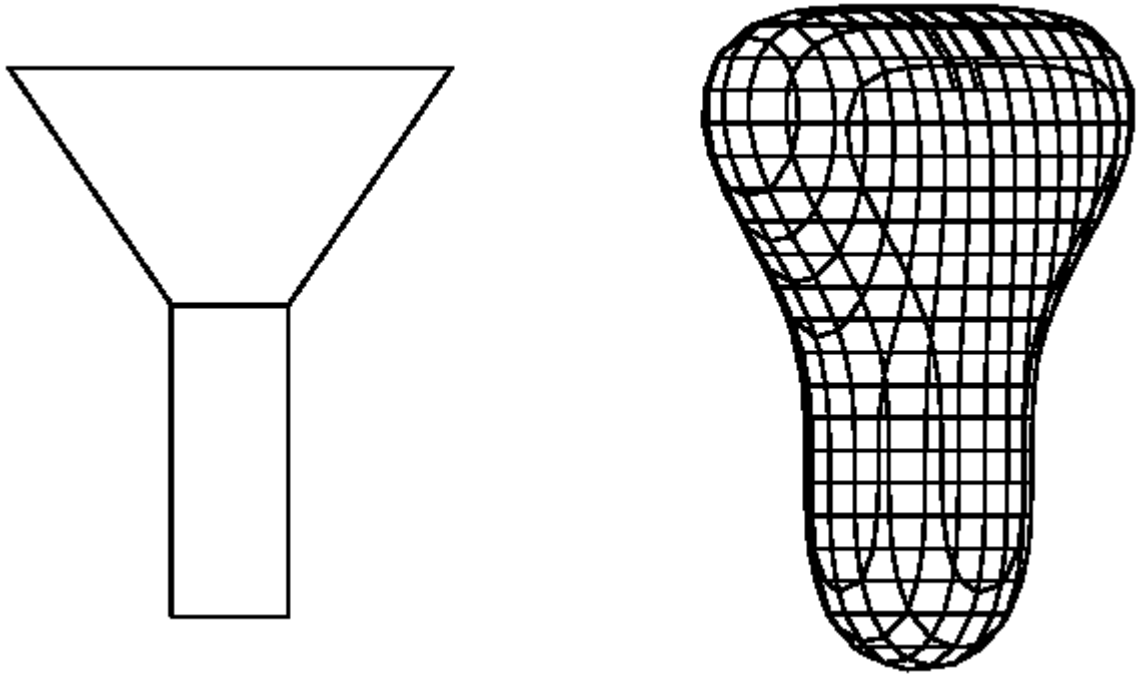




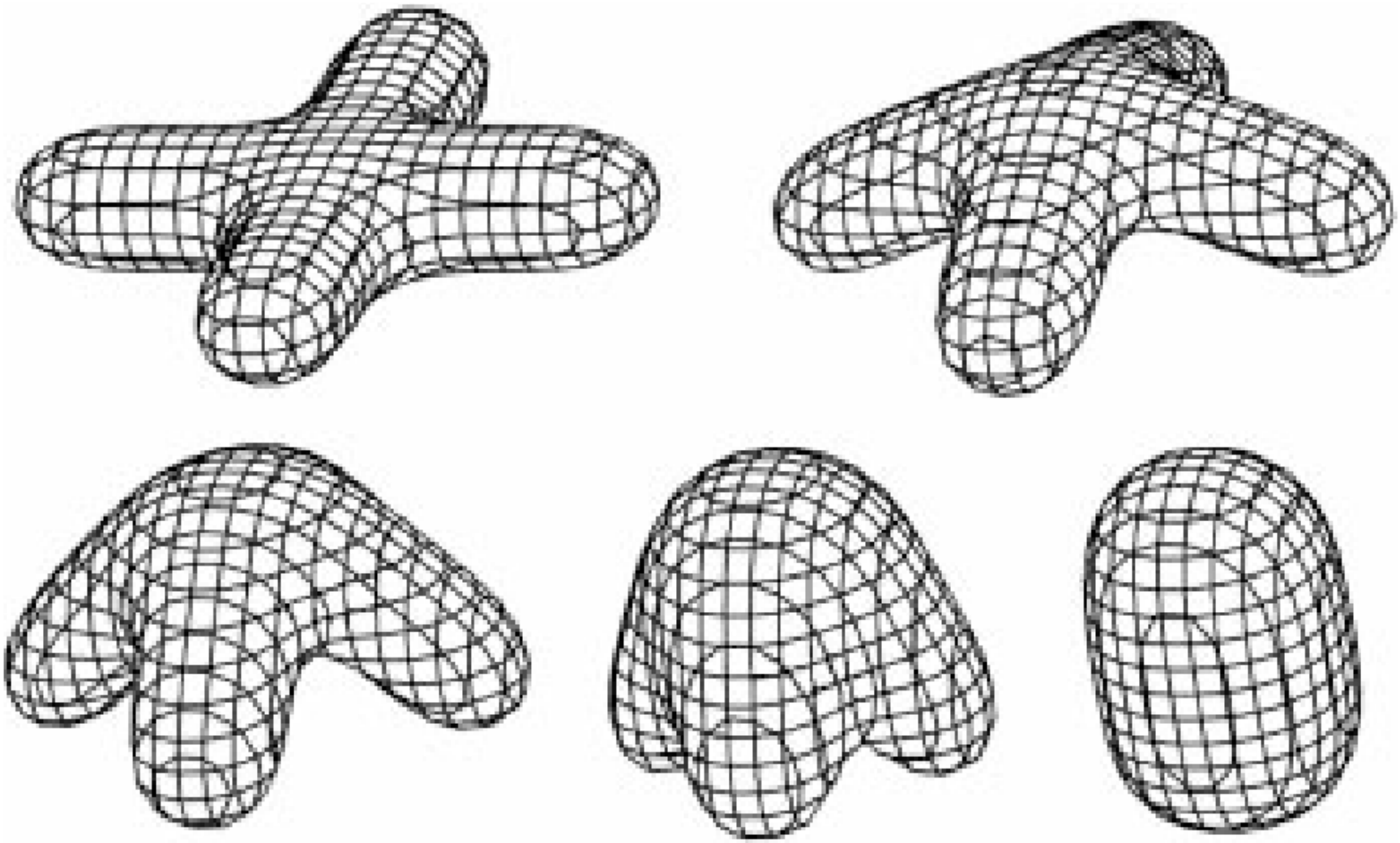
Two Line Segments
and their Convolution



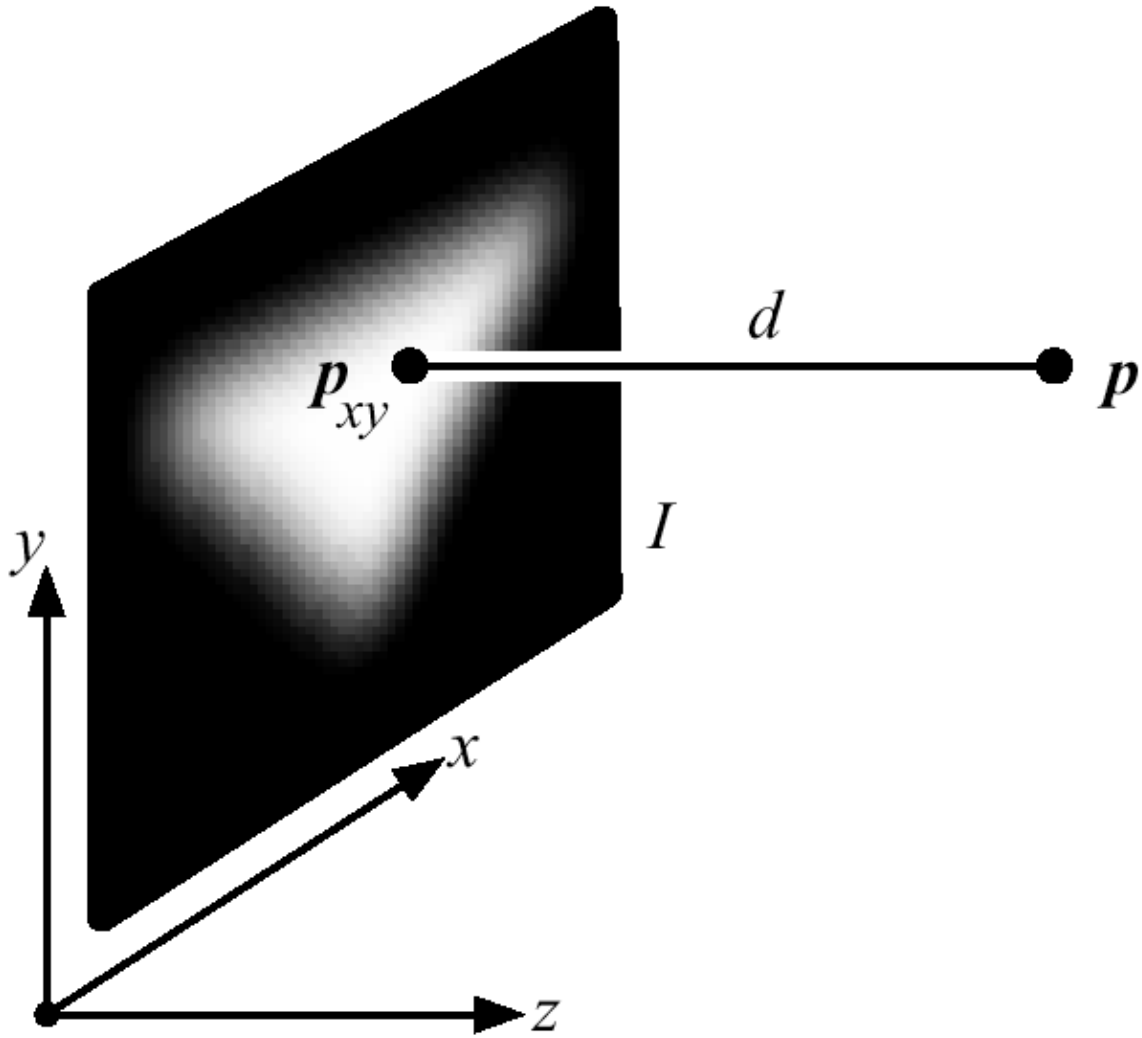
A Simple Convolution Surface



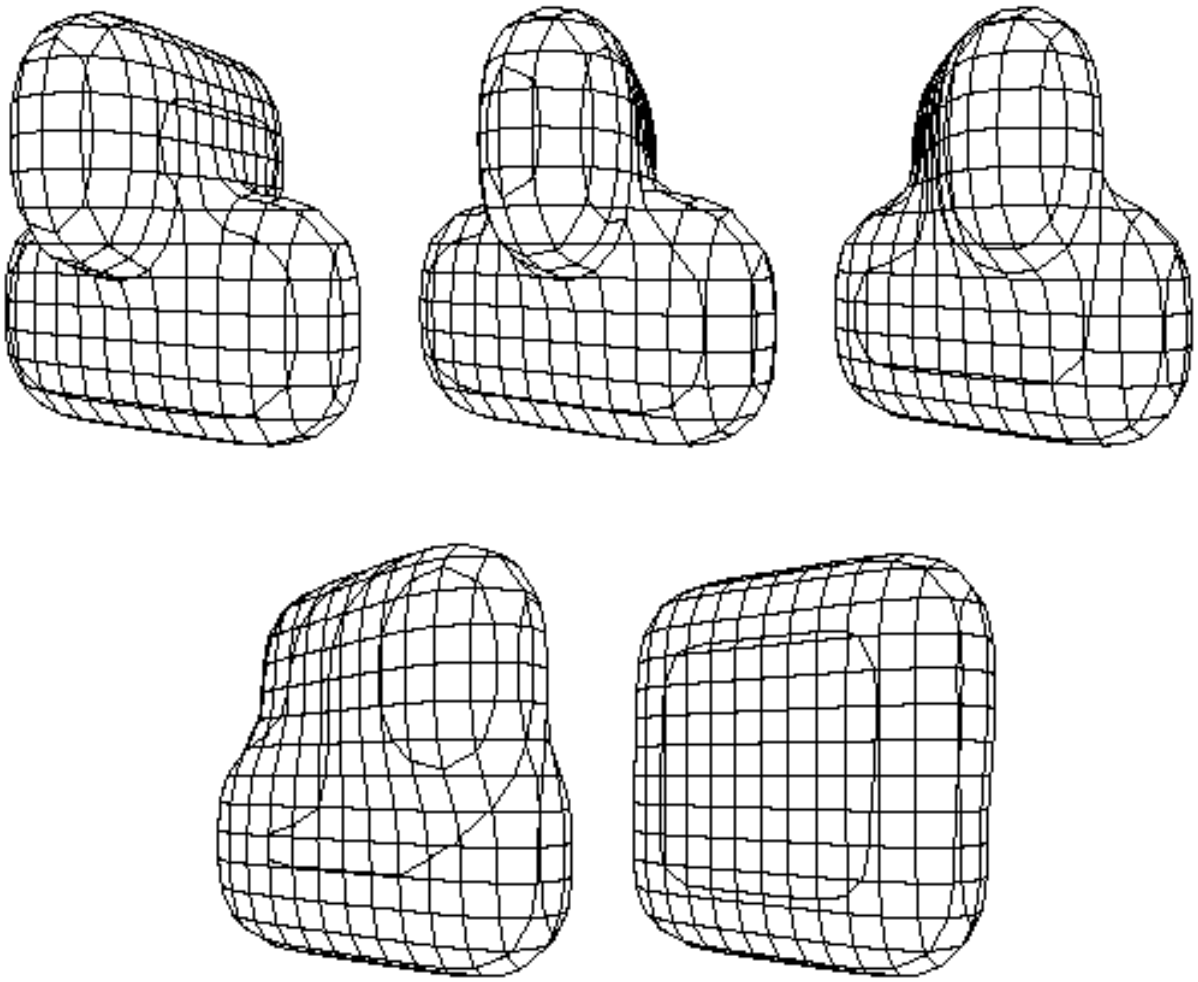
Folding Convolution Surface



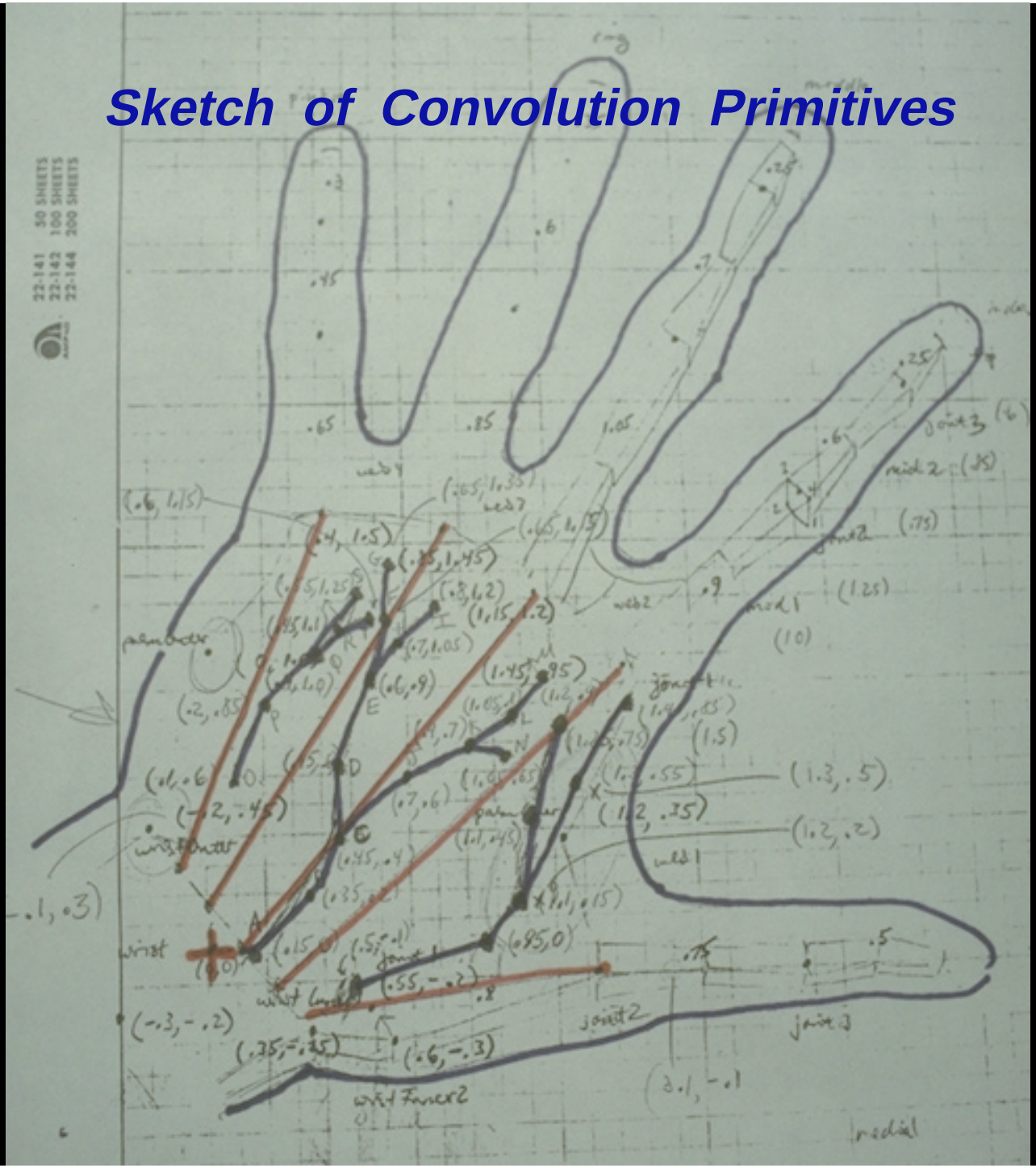
Evaluating 3D Convolution



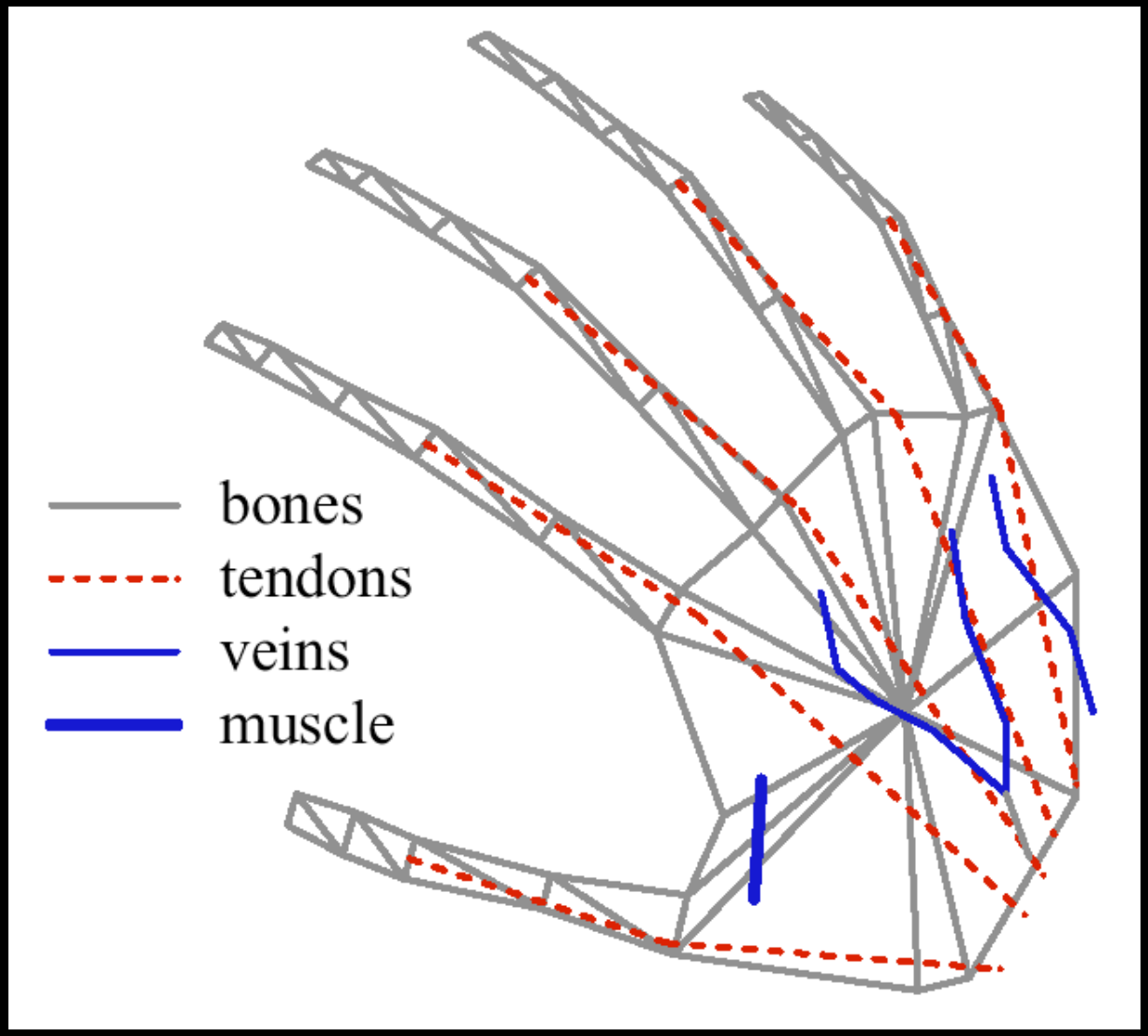
Convolution of Rotating Rectangles



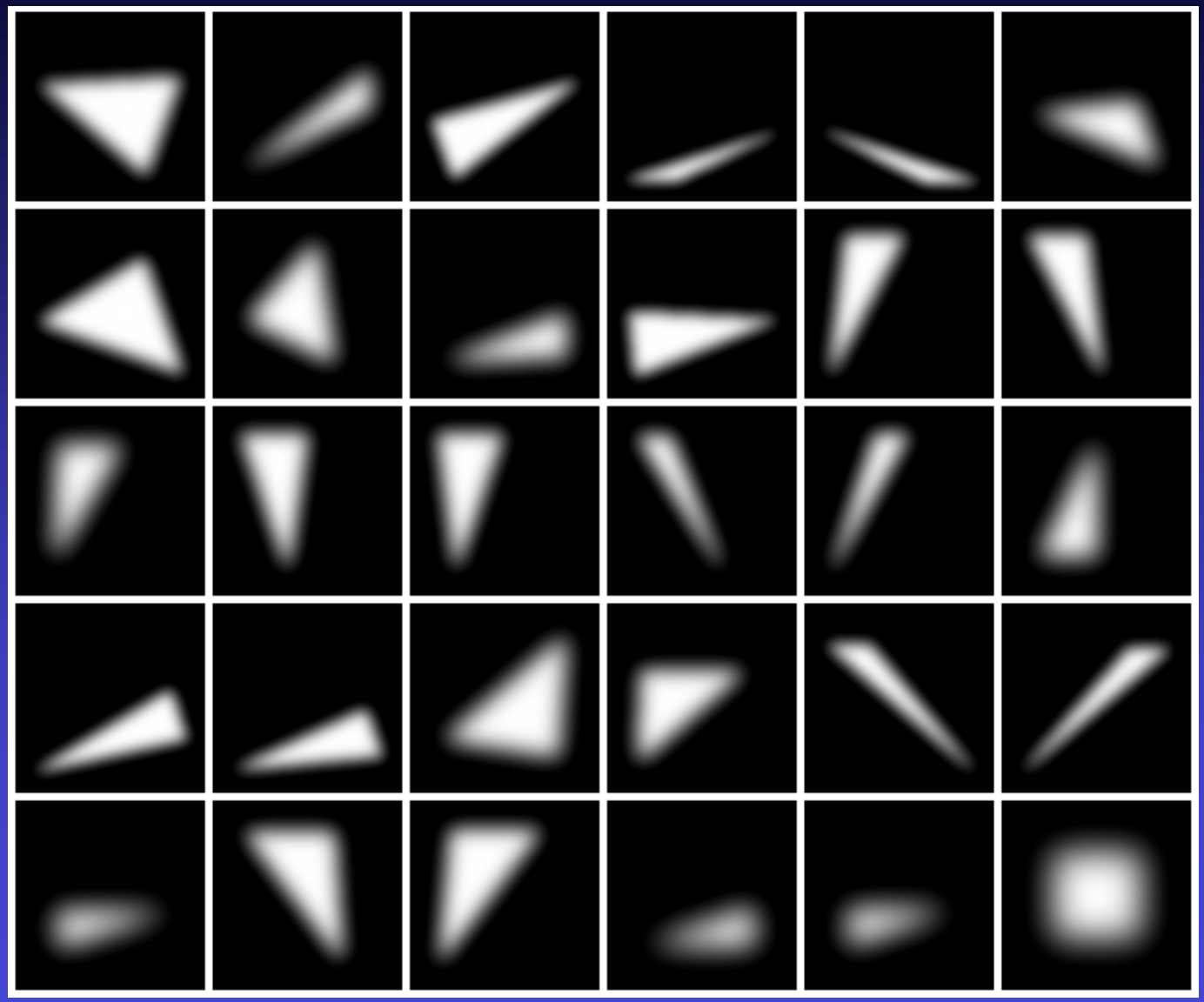
Sketch of Convolution Primitives

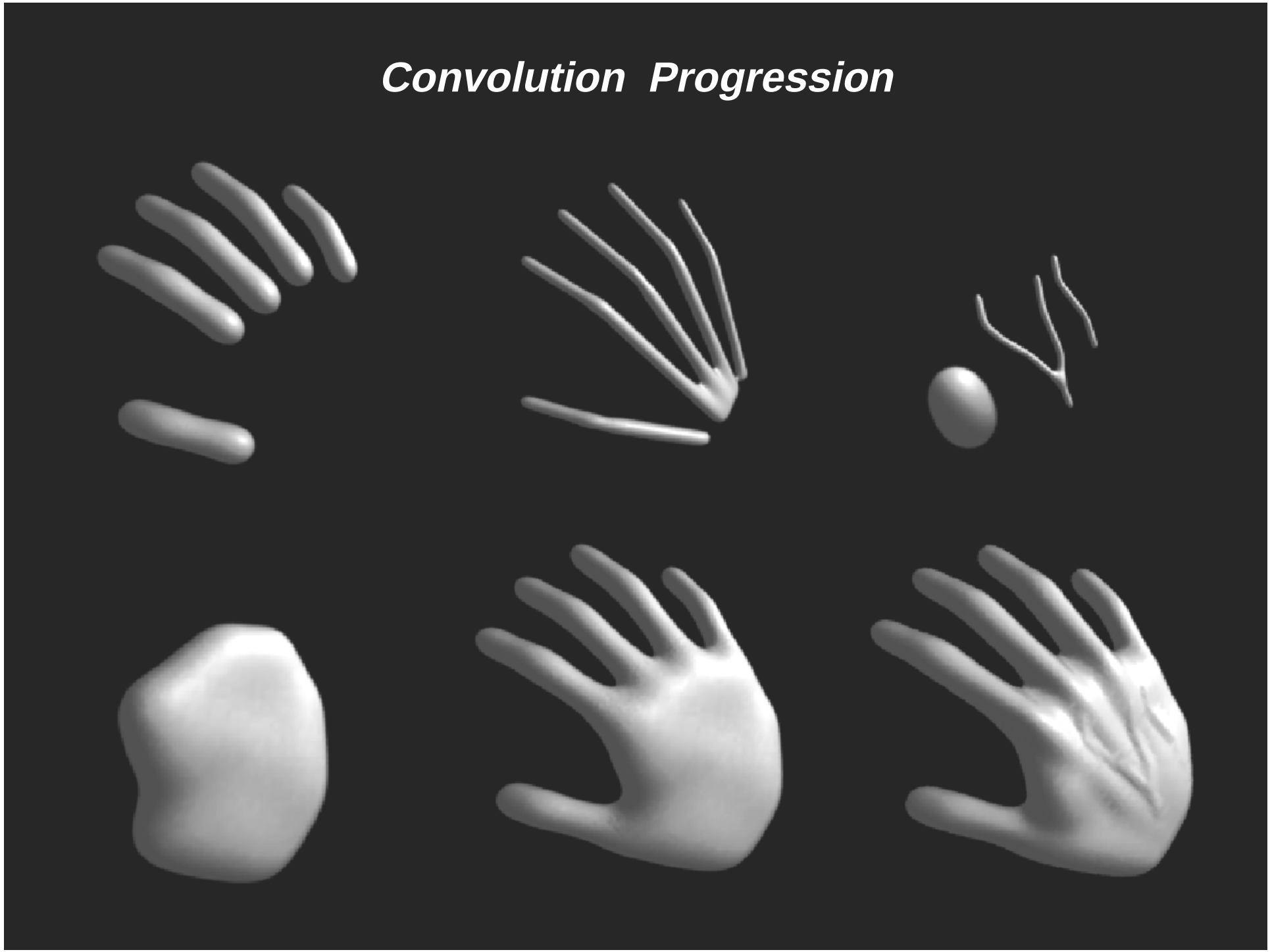


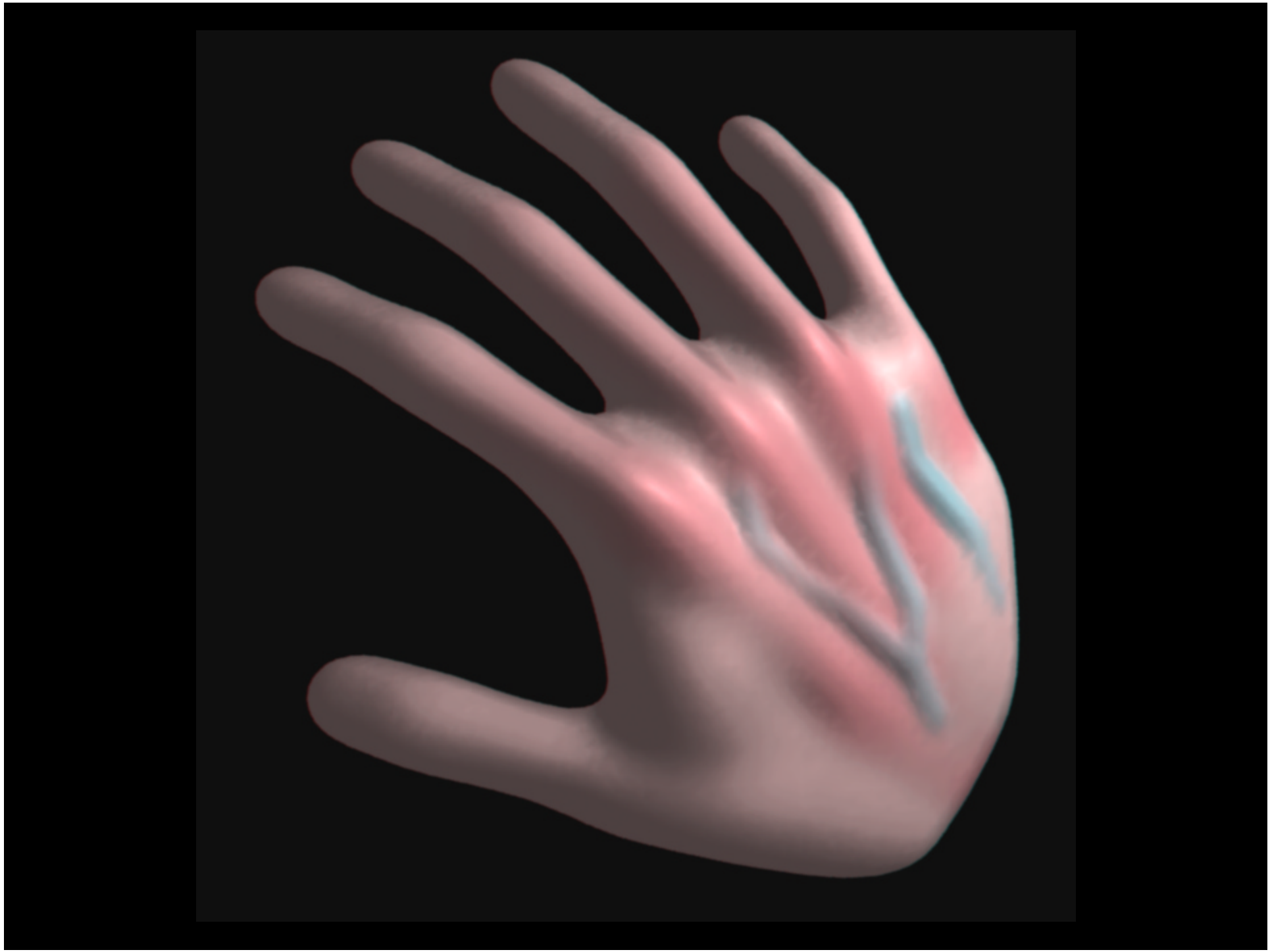
Convolution Primitives



Convolutions in 2D



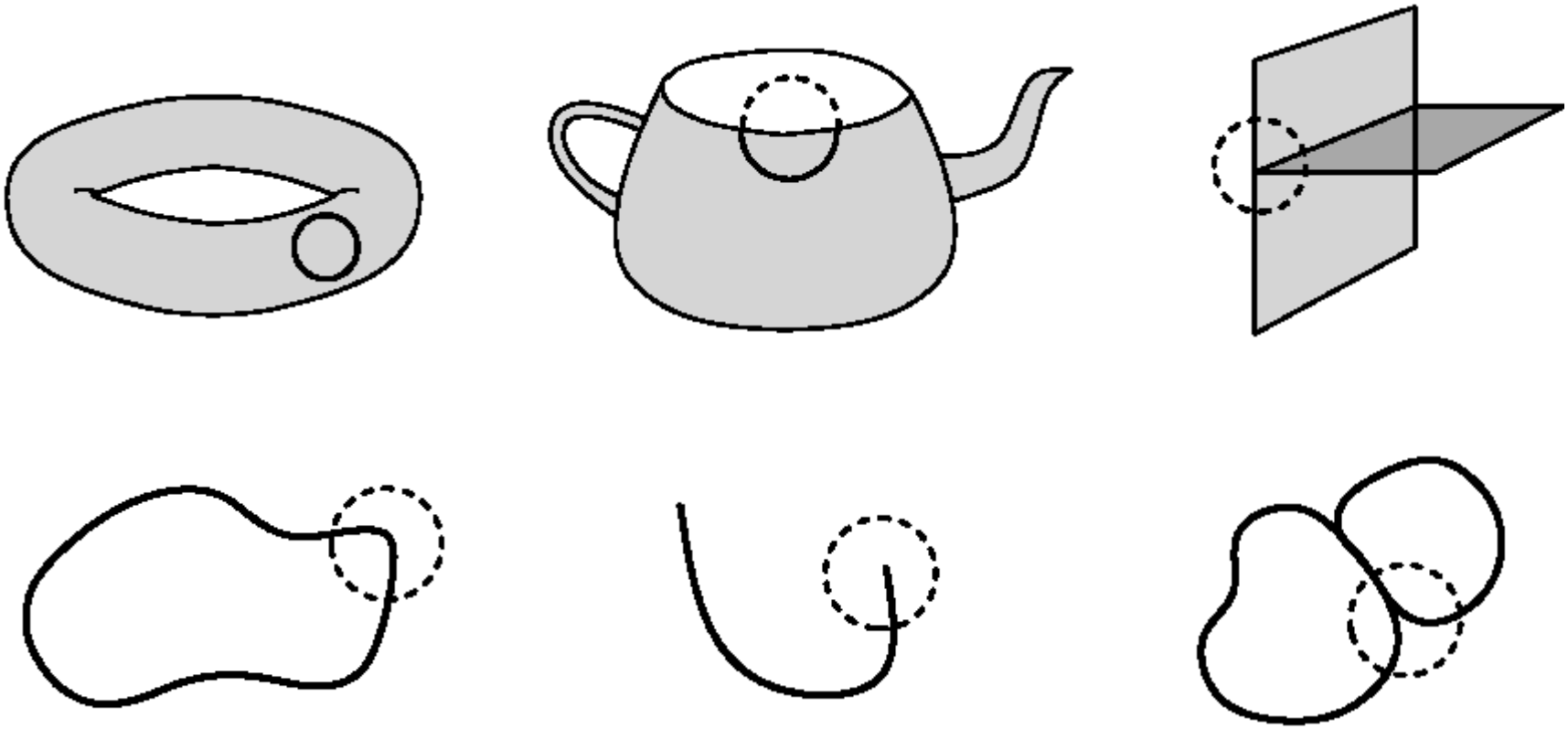

Convolution Progression



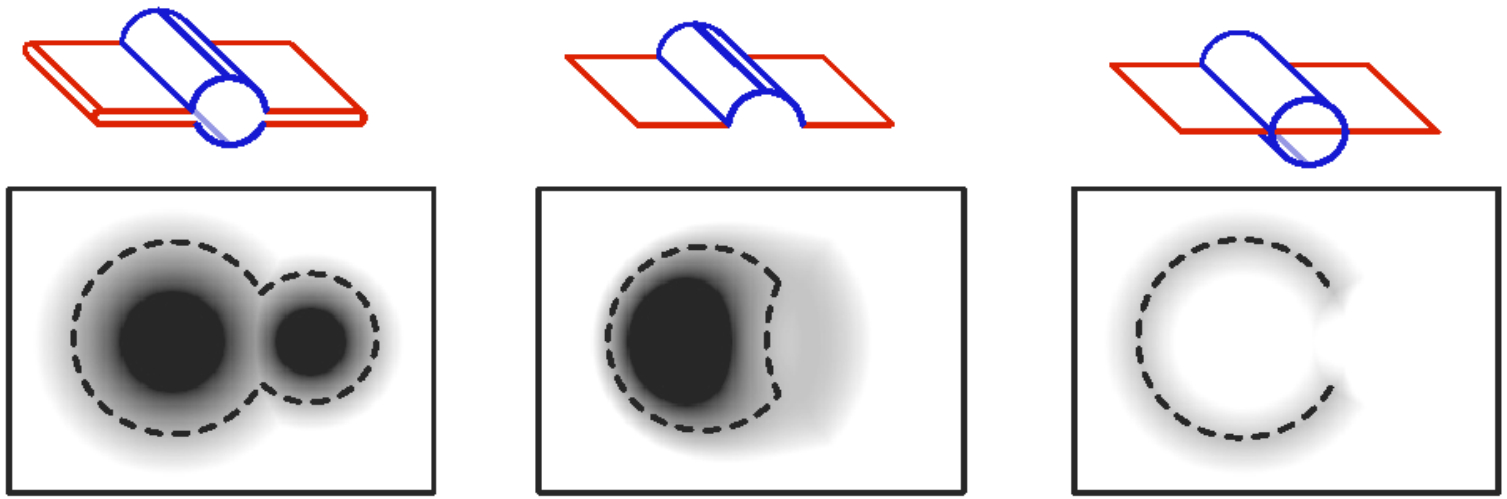

‘blobby’

Non-Manifold Leaves

Manifold, Manifold-with-Boundary, and Non-Manifold Surfaces and Contours



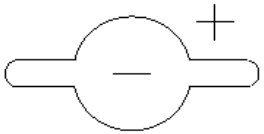
Non-Manifold Schemes

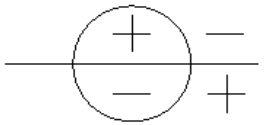


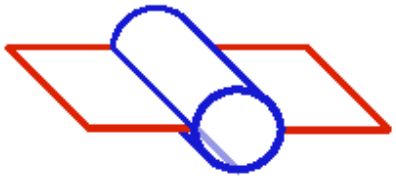
min (f_1, f_2)

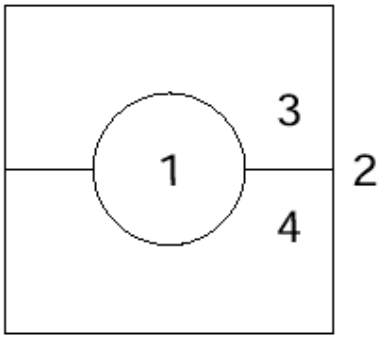
f_1 *max*(f_1, f_2)

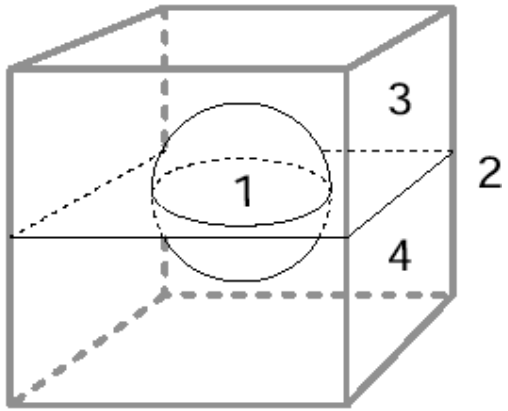
abs (f_1)-*min* (0, f_2)



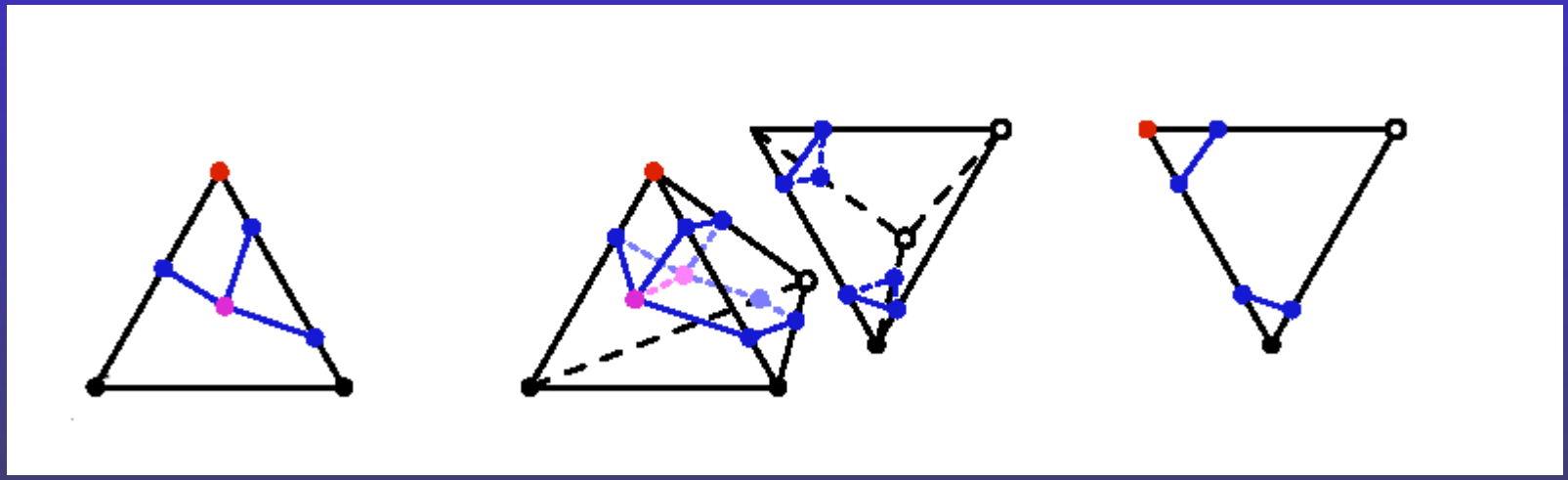




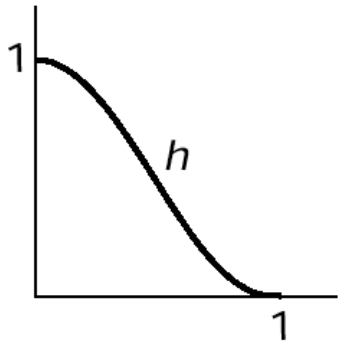


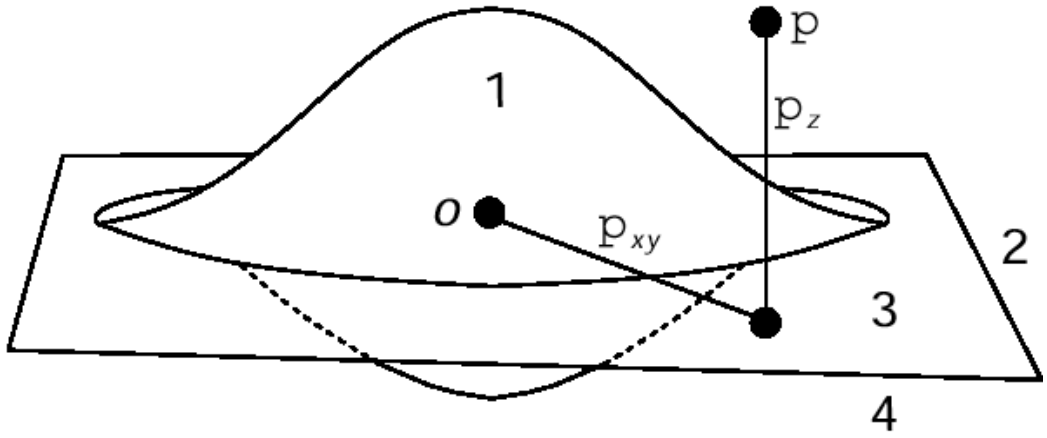


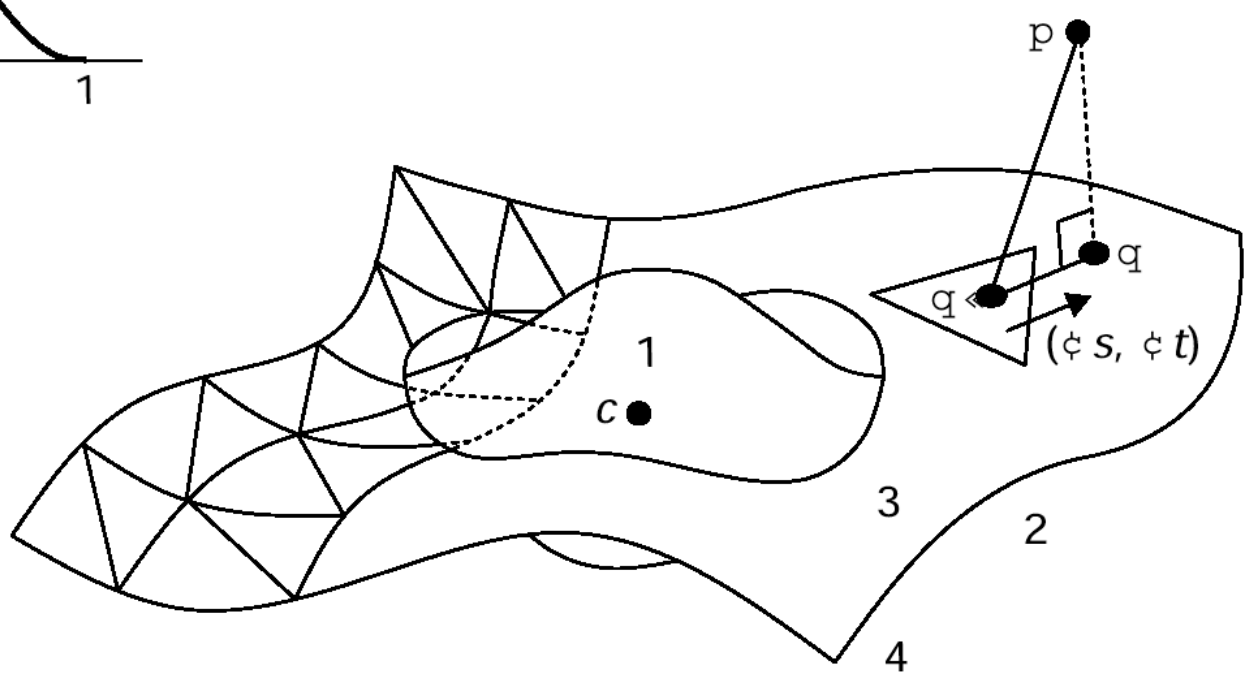
Non-Manifold Polygonization Complications



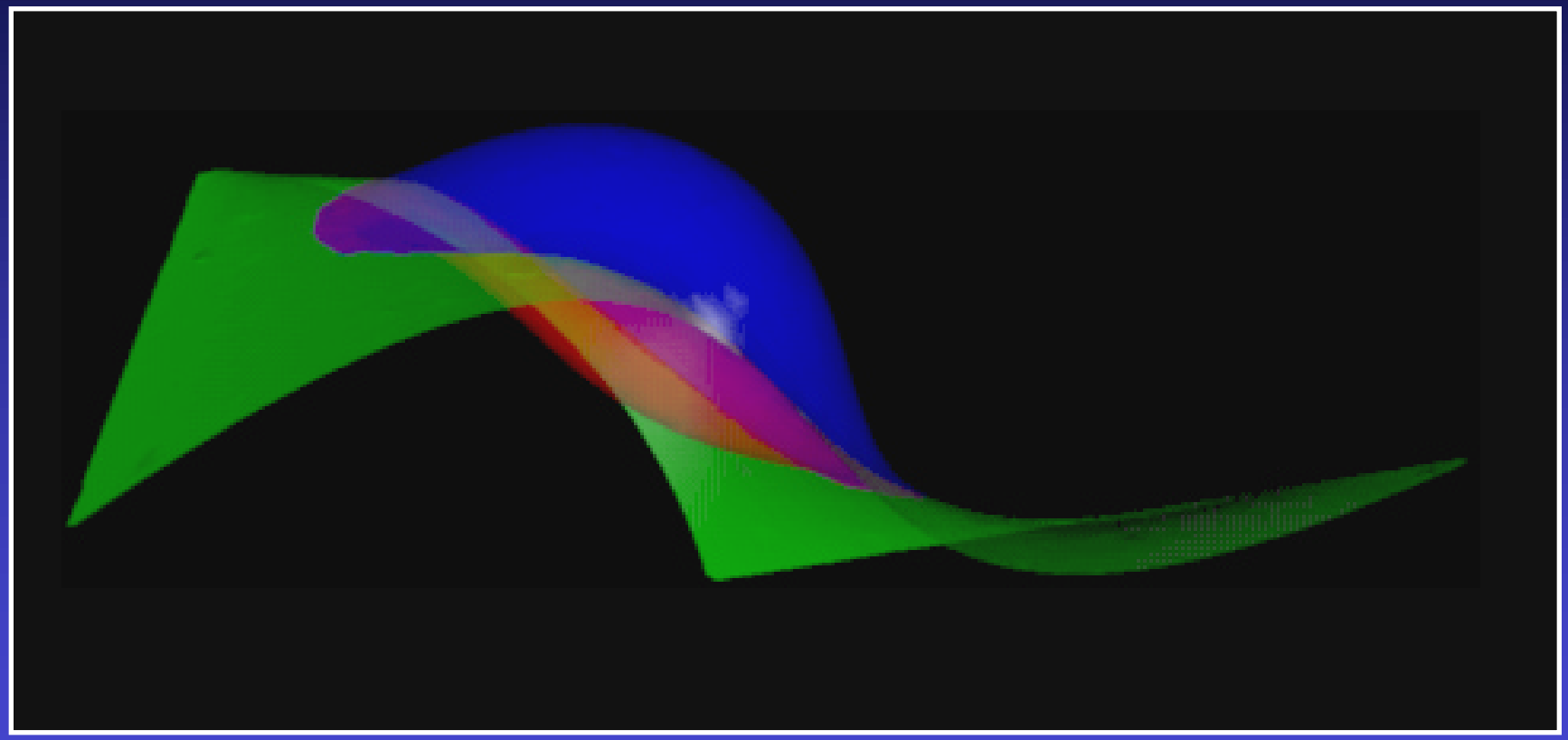
Non-Manifold Blend to Plane and Patch





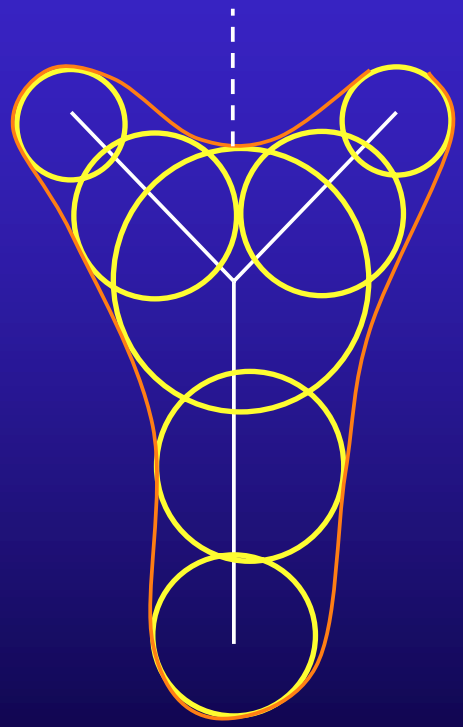


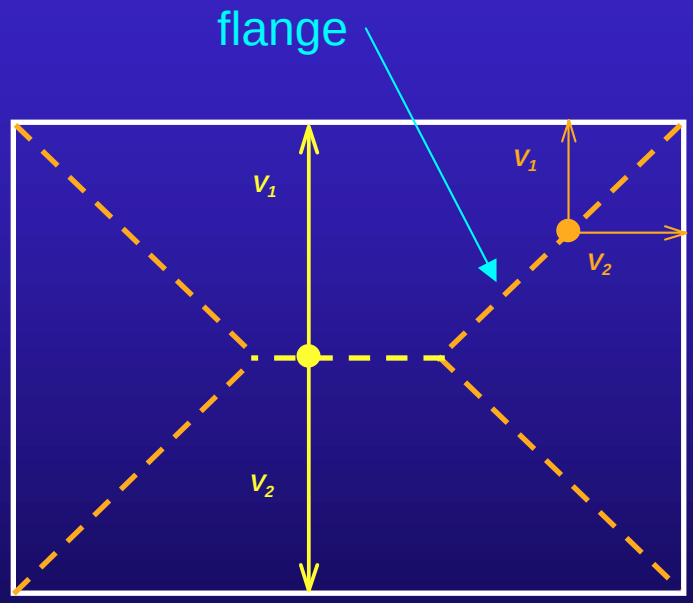
Non-Manifold Implicit Blend



The Medial Axis (Surface)

the locus of centers of maximally sized inscribable spheres (circles)

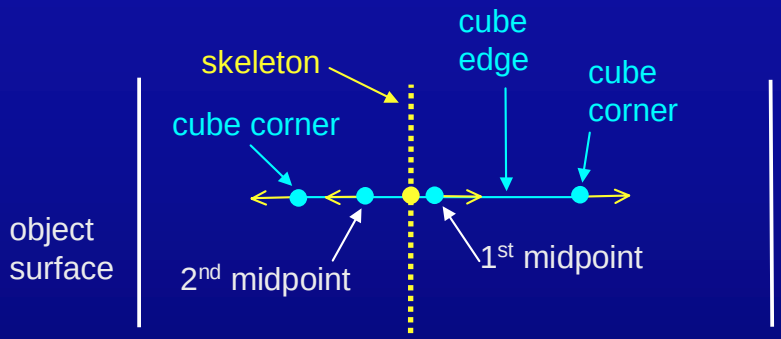




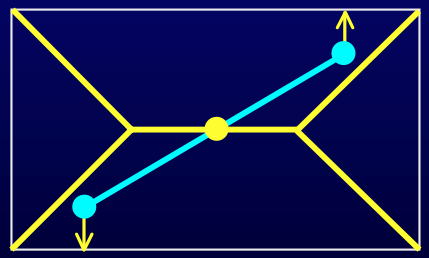
Skeletonization: Object → Locus

Reconstruction: Locus → Object

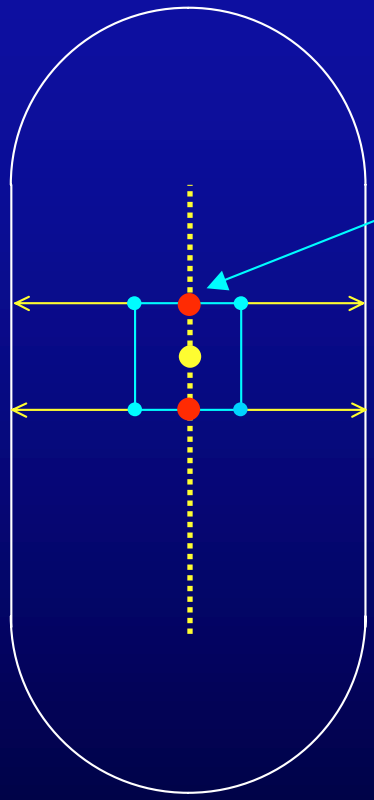
Continuation Details



1) convergence along edge



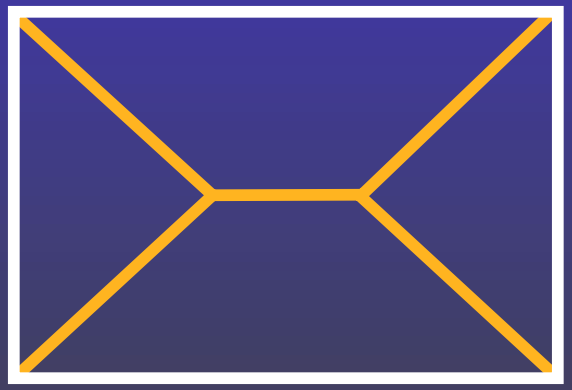
2) finding first skeletal point

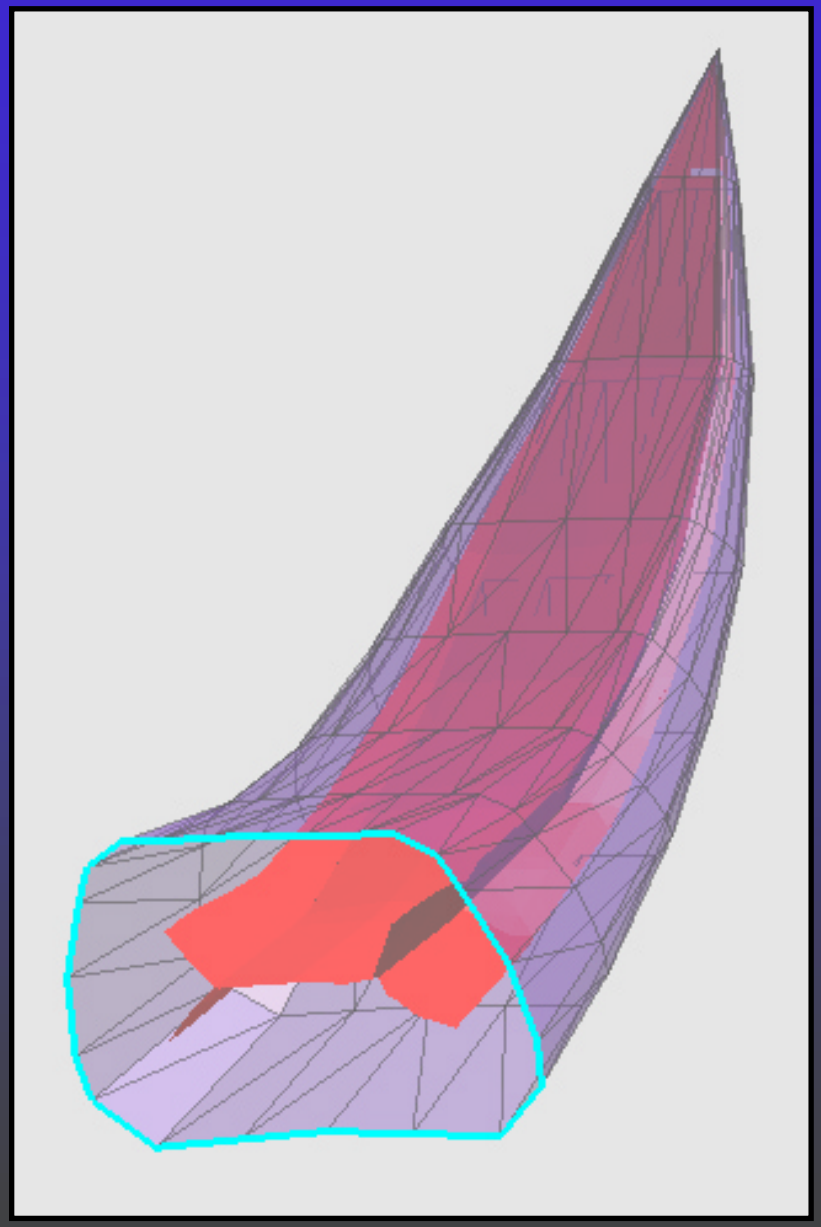


3) initial skeletal point surrounded by initial cell

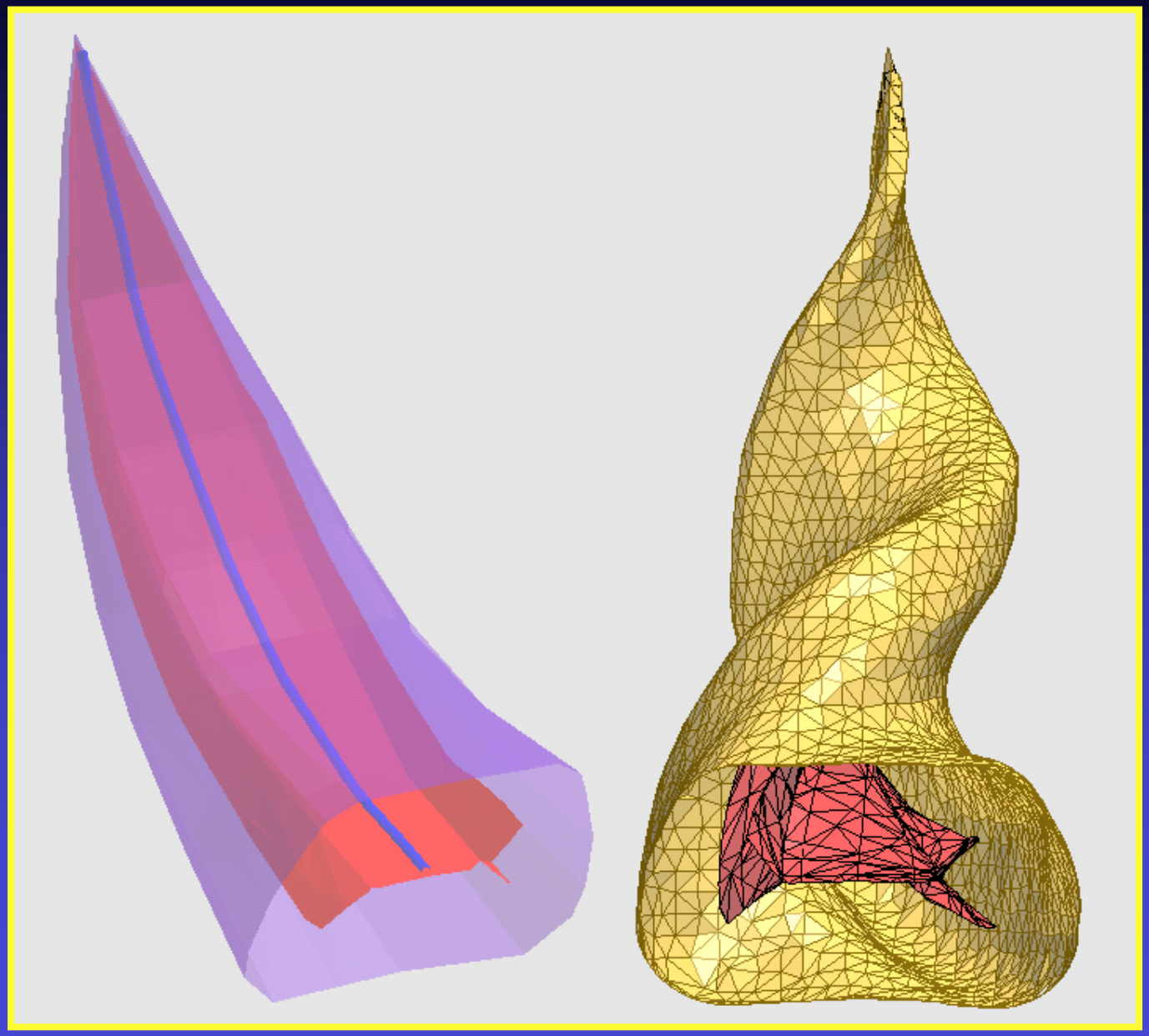
2D and 3D Medials

- **geometrically derivable**
- **stick-figure control**





Medial Axis/Surface

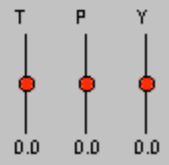


Original is *re-constructable*

distance surface **(exact reconstruction)**

convolution surface **(differentiable)**

Animation



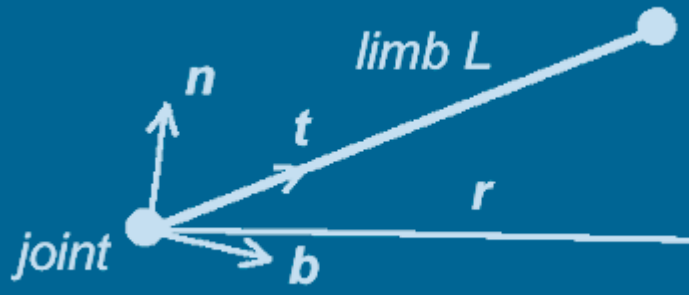
Reconstruction

non-interactive
prone to artifacts
variable vertex count

Deformation (vertex blending)

inherent artifacts
sensitive blend weights

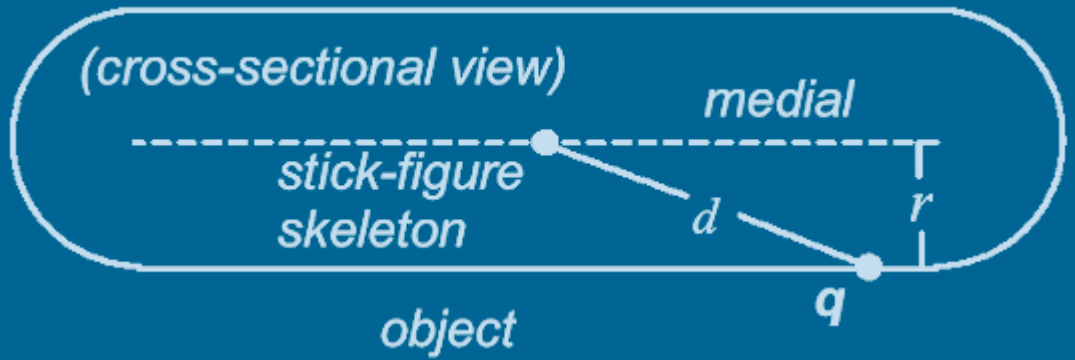
Medial-Based Vertex Deformation



frame *F* for limb *L*

$$q' = \sum w_i (F_p + q_x F_t + q_y F_n + q_z F_b))$$

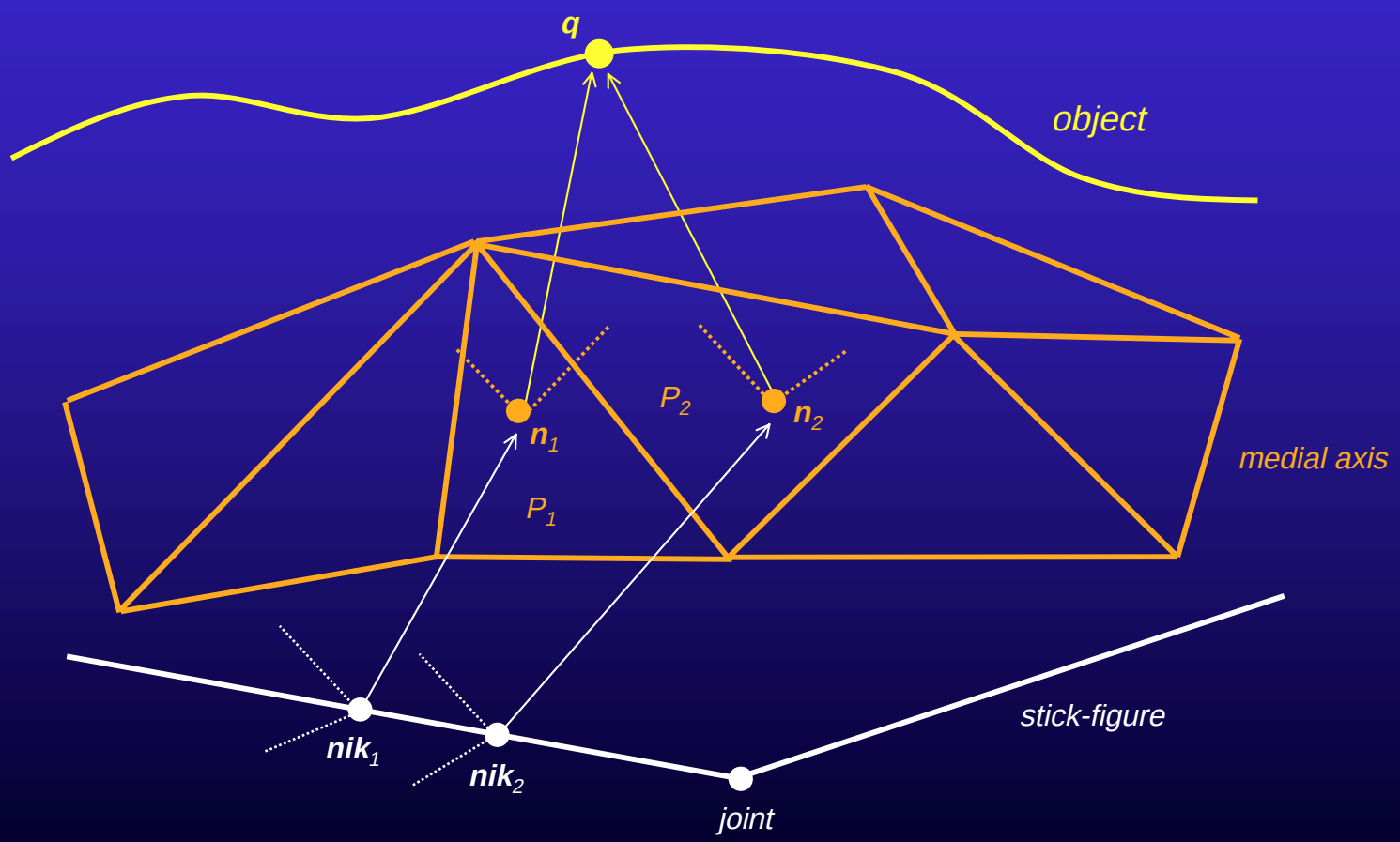
distance *d* is poor estimate of thickness *r*

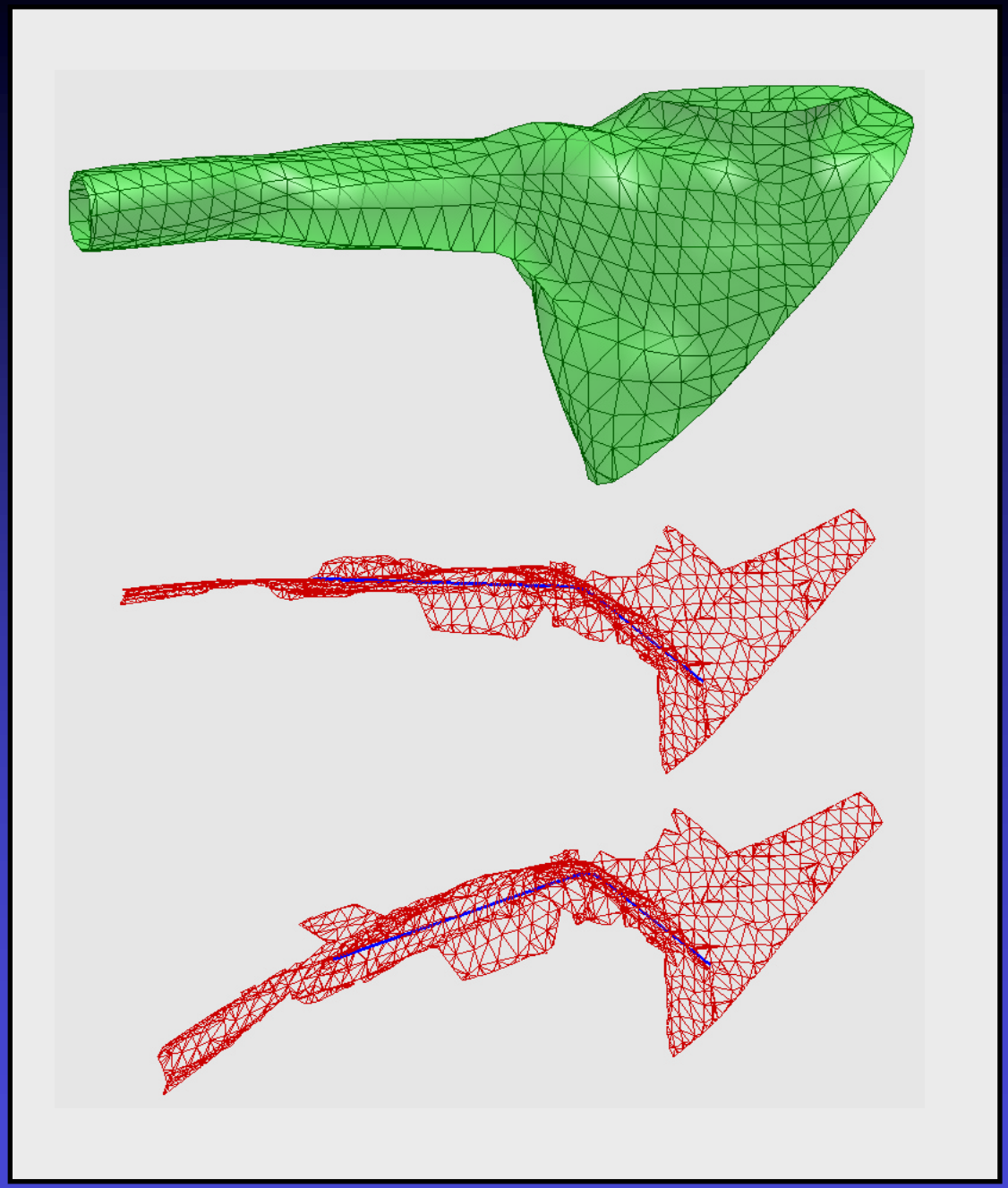


Vertex Deformation

as a weighted vector sum

using the medial axis



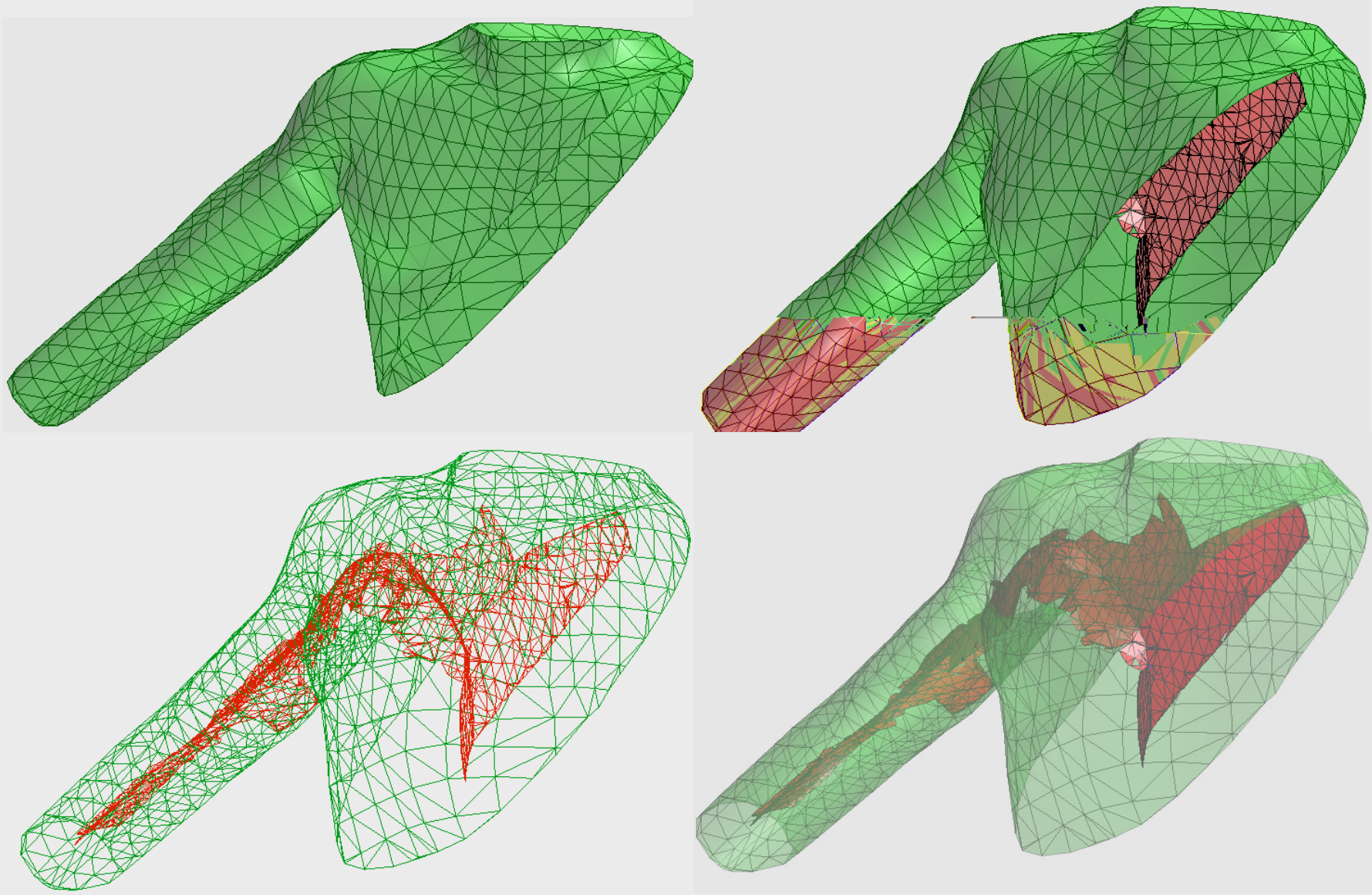


Vertex Deformation

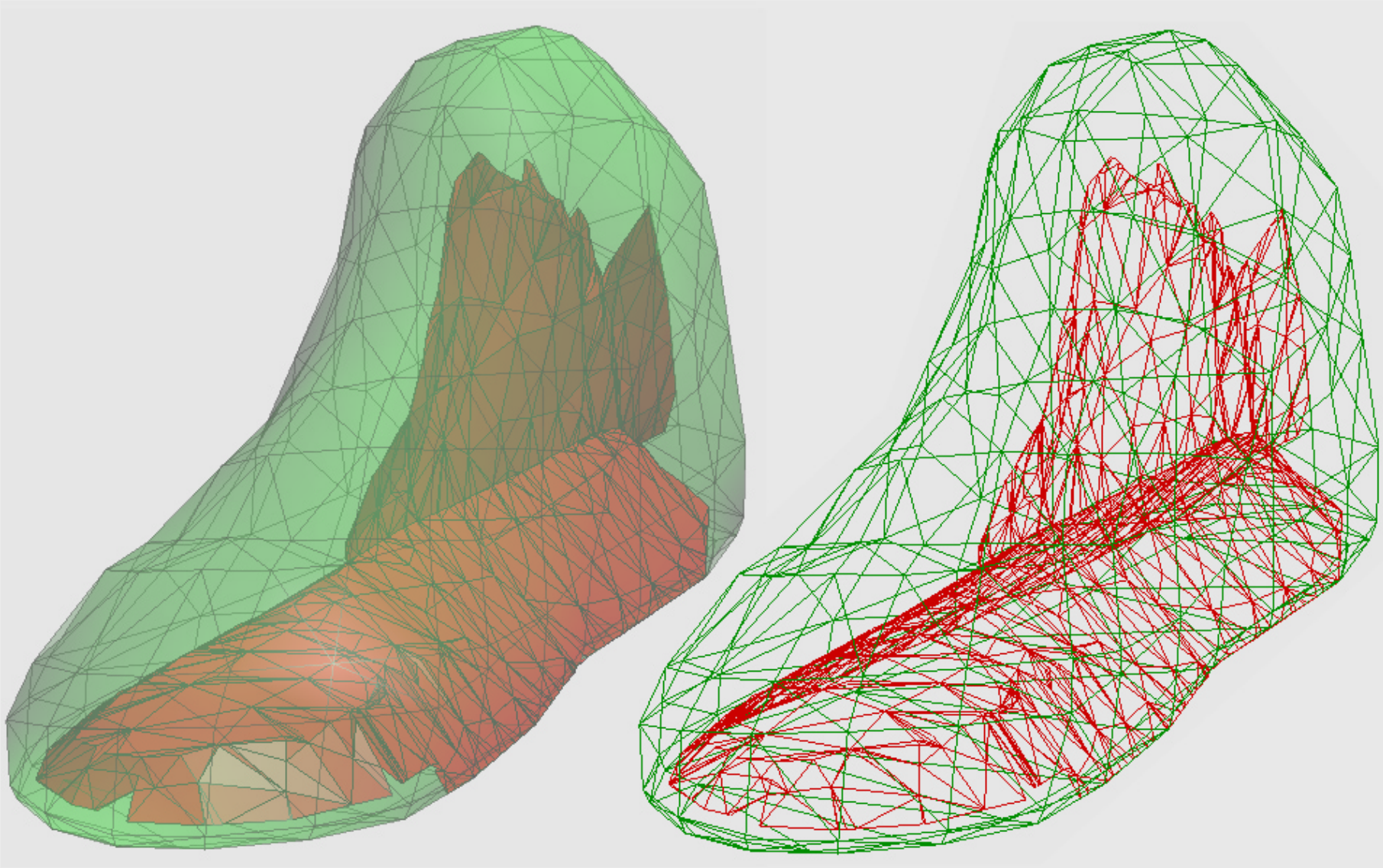
IK-Skeleton Manipulates Medial

Torso and Medial Axis

(various displays)

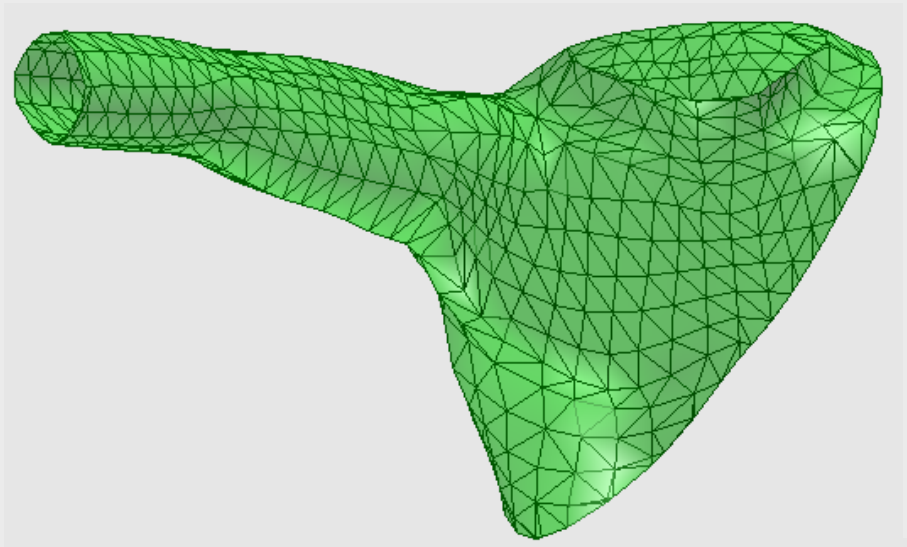


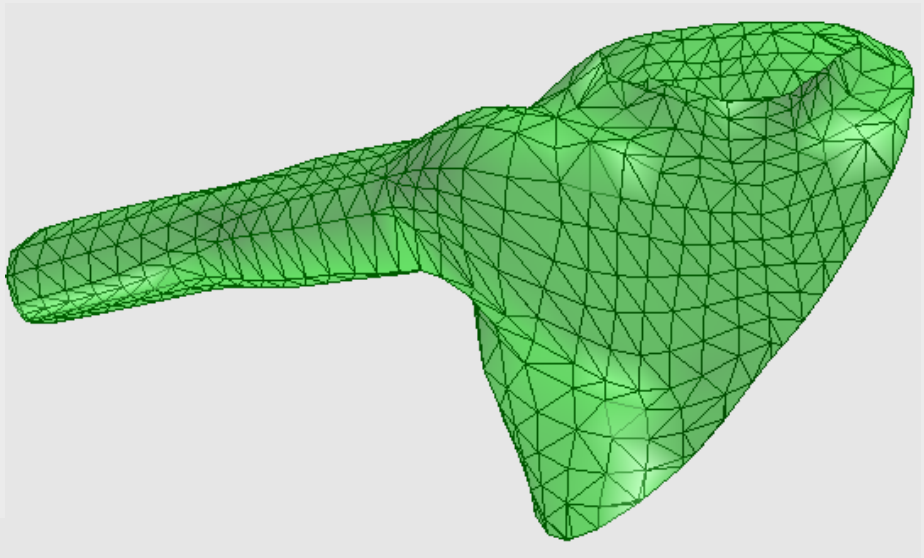
Foot and Medial Axis

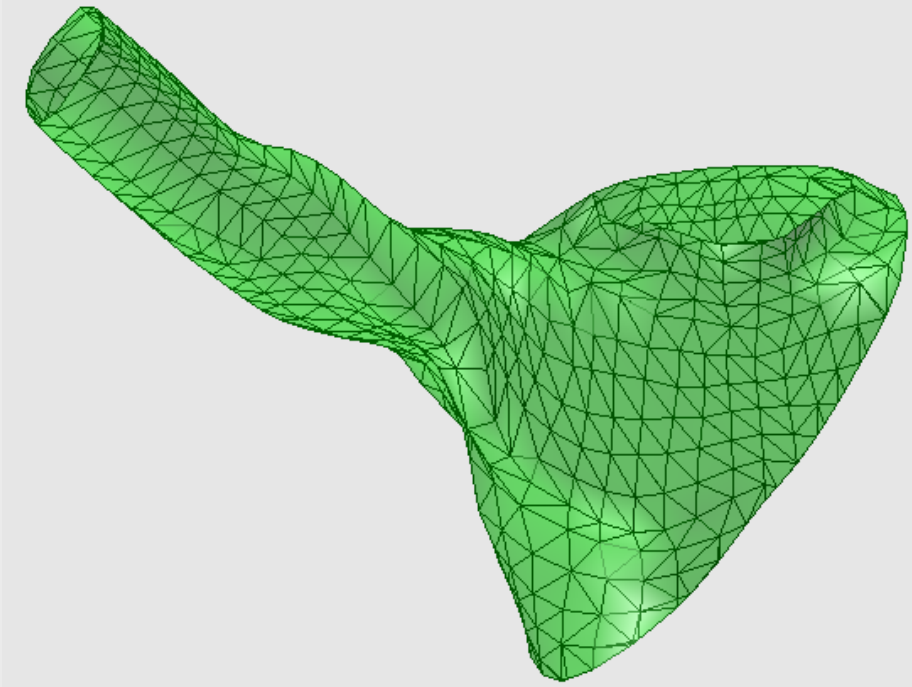


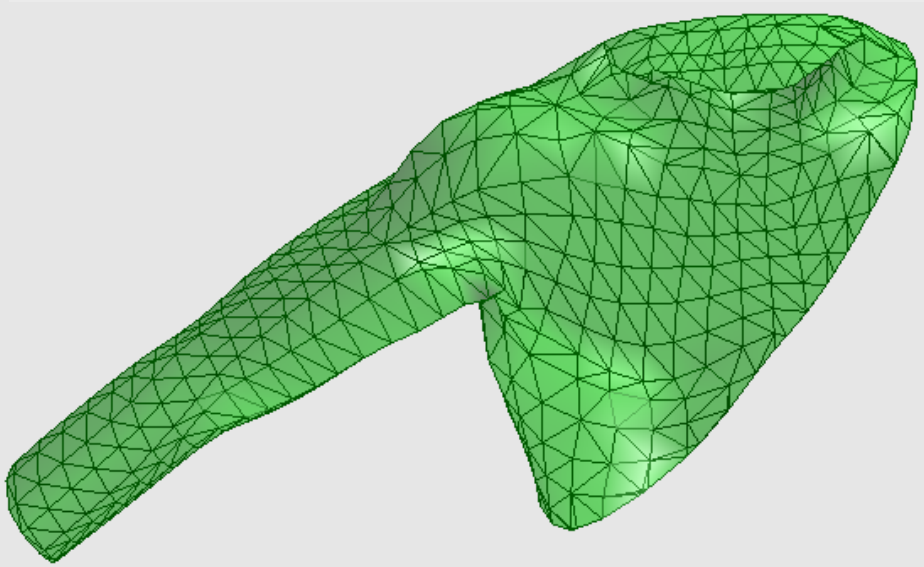
Demo

Object Articulation by Vertex Deformation









Questions?